Scientific Computing

Wed, Feb 25

Announcements

* HW 3 is due Friday!
  covers brute force, search spaces,
  divide + conquer

* Midterm exam
  Monday, March 2
  in class portion + takehome portion
  due Friday, March 6

* I posted a "HW 4 preview" in the D2L Dropbox for you to practice BT + BB.

Office Hours:

Mon, 9:30-10:30
Fri, 2:00-3:00

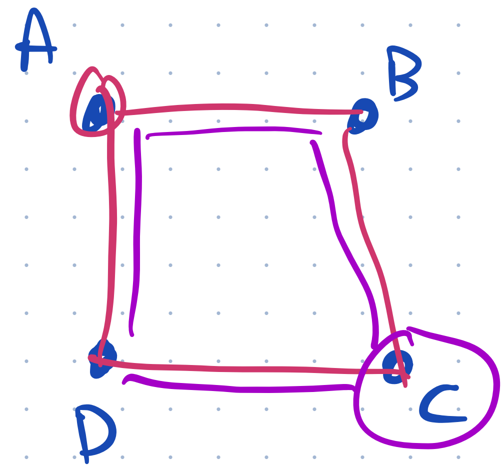
Cudahy 307

One more example – traveling salesman

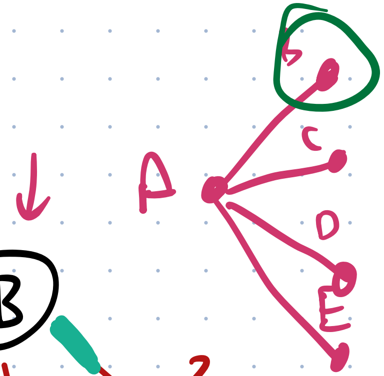
* Pick a start city C_1. (assume 4 cities)

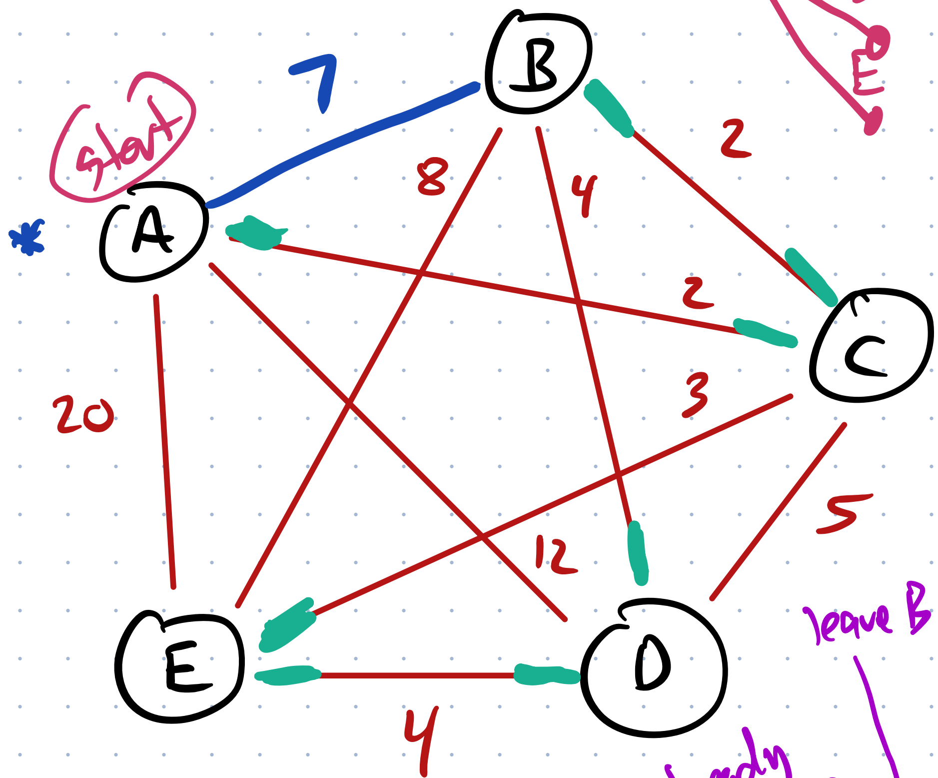
  Branch: Next city to visit that hasn't been visited yet.

- C_1
  - C_2
    - C_3 — C_4
    - C_4 — C_3
  - C_3
    - C_2 — C_4
    - C_4 — C_2
  - C_4
    - C_2 — C_3
    - C_3 — C_2



* Lower bound:





We're going to have to exit B. Cheapest way: 2.

We're going to have to enter and exit C. Cheapest: 4

D: 8

E: 7

Back into A: 2

already chosen

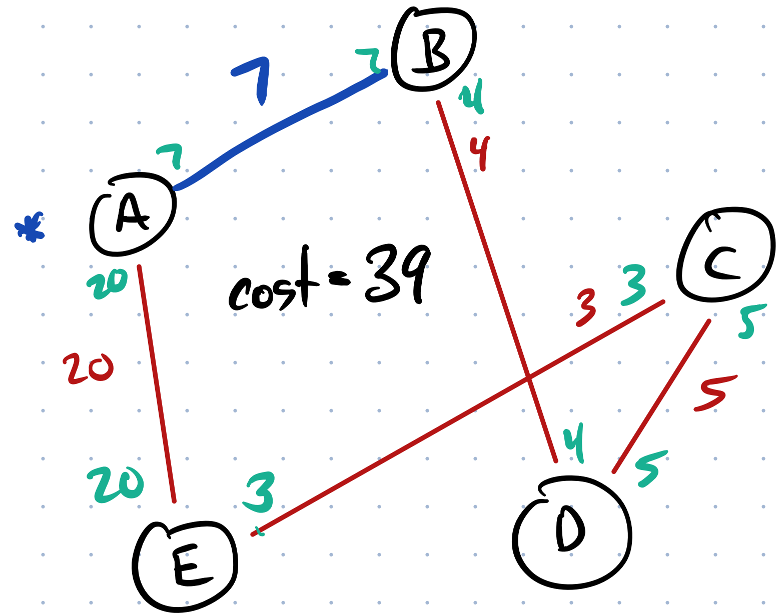
leave B

Wrong: lower bound = $\underbrace{7}_{A\to B}+2+\underbrace{4}_{C}+\underbrace{8}_{D}+\underbrace{7}_{E}+\underbrace{2}_{\text{return } A} = 30$

Double counts! When you exit B, you enter some other node. When you exit C, you enter some other node, etc.

Let T be a given tour (a solution).

If you add up the cost going into and out of each city, you get double the cost, because you're counting each edge twice.



$$\text{cost}(T) = \frac{1}{2} \cdot \sum_{v \in V} \big([\text{cost to enter } v] + [\text{cost to exit } v] \big)$$

sum over each vertex v

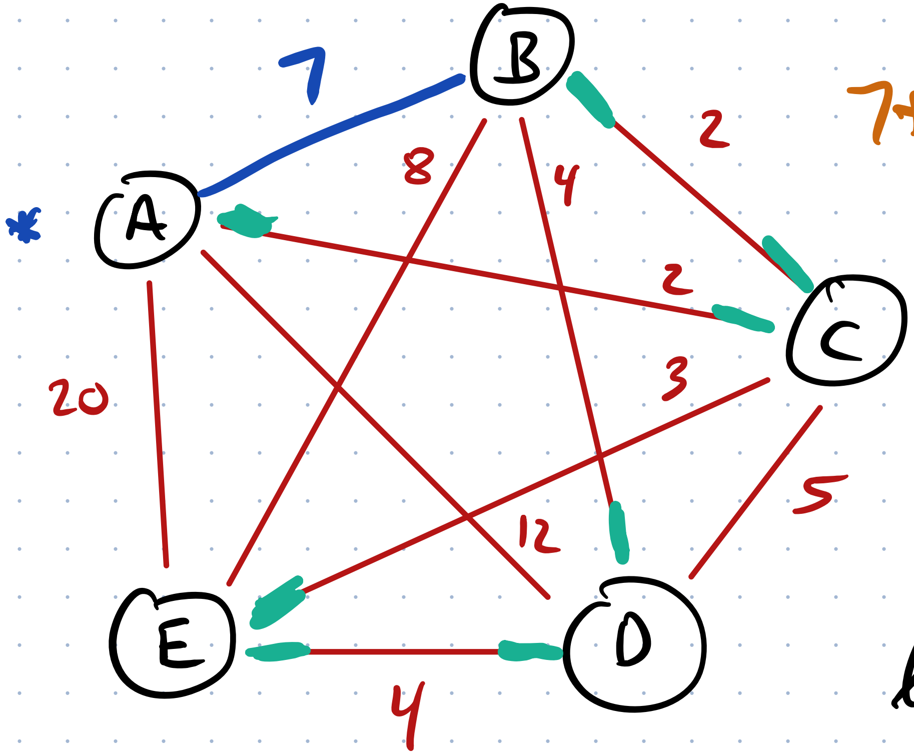
Now suppose we're in some subspace S and we want a lower bound on the cost of any tour in S. Let $T \in S$ be arbitrary.

↳ a tour that starts $A \to B$

$$\mathrm{cost}(T) = \frac{1}{2}\Big(\underbrace{[\text{enter } v_1] + [\text{exit } v_1]}_{\substack{\geq \text{ sum of two} \\ \text{cheapest edges} \\ \text{attached to } v_1}} + \cdots + \underbrace{[\text{enter } v_n] + [\text{exit } v_n]}_{\substack{\geq \text{ sum of two} \\ \text{cheapest edges} \\ \text{attached to } v_n}}\Big)$$

$+ \cdots +$

$\geq \frac{1}{2}\Big($sum of: for each vertex, use any edges already decided on, plus cheapest remaining, to get two total$\Big)$

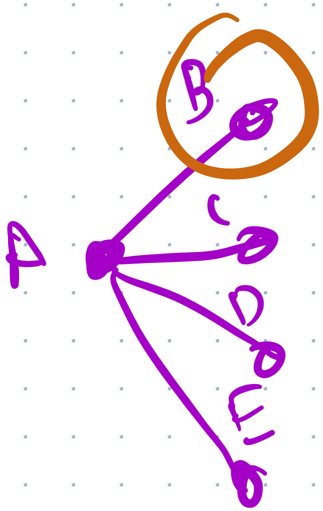


chosen

$$7 + \frac{1}{2}\left(\underbrace{\cancel{7}+2}_{A} + \underbrace{\cancel{7}+2}_{B} + \underbrace{2+2}_{C} + \underbrace{4+4}_{D} + \underbrace{4+3}_{E}\right)$$

$$= \frac{1}{2}(37) = 18.5$$

lower bound: 19



B+B tree time!

Greedy: A → C → B → D → E → A
$= 2+2+4+4+20 = 32$

Another: B → C → A → D → E → B
$= 2+2+12+4+8 = 28$

C → A → B → D → E → C
$= 2+7+4+4+3 = 20$!