MATH 2100 / 2105 / 2350 – HOMEWORK 4

due Wednesday, March 27, at the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Prove that if 3 divides $4^{n-1} - 1$ then 3 divides $4^n - 1$.
2. Prove that for all positive integers $n \geq 4$,
$$n! > 2^n.$$
3. Prove that for all positive integers n, the number $5^{2n+1} + 2^{2n+1}$ is divisible by 7.
4. Prove that for all positive **odd** integers m, the number $m^2 - 1$ is divisible by 8.
5. Prove that for all positive integers n,
$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$
You may use the theorem we proved from class that says
$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$
6. Prove that for all positive integers n,
$$\sum_{k=0}^{n} (k \cdot k!) = (n+1)! - 1.$$
7. Prove that for all $n \geq 0$:
$$\sum_{i=0}^{n} i2^i = 2 + (n-1)2^{n+1}.$$
8. Prove that for all $n \geq 1$:
$$1 + 3 + 5 + \cdots + (2n-1) = n^2.$$