MATH 2100 / 2105 / 2350 – HOMEWORK 3

due Wednesday, March 6, at the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Read both of the handouts from class on February 25 (they're also posted on our website) and write yourself a page of bullet points for things to remember when writing proofs. Keep it somewhere safe, and bring it out every time you write a proof for the next few weeks so you can skim it over and make sure you've followed your bullet points. On your homework assignment, just tell me whether or not you did this, and include any thoughts you have about it. This one is on the honor system.

2. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

   For all integers m, 6 divides $2(m^3 - m)$.

3. Prove that if n^2 is divisible by 5 then n is divisible by 5.

4. Suppose a, b, and c are integers. Prove that if a divides b and b divides c, then a divides c.

5. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

   If n is a positive even integers, then $3^n + 1$ is divisible by 5.

6. Prove that for any integer n, if n is even then $n^3 + 2n$ is divisible by 4.

7. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

   If n is a positive even integer and $n \geq 4$ then $2^n - 1$ is not prime.

8. Prove by induction that for all positive integers n,

   $$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$

   You may use the theorem we proved from class that says

   $$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

9. Prove by induction that for all positive integers n,

   $$\sum_{k=0}^{n} (k \cdot k!) = (n+1)! - 1.$$