MATH 2100 / 2105 / 2350 – EXAM 1

Wednesday, February 20

Name: Key

Instructions: Please write your work neatly and clearly. **You must explain all reasoning. It is not sufficient to just write the correct answer.** You have 75 minutes to complete this exam. You may not use calculators, notes, or any other external resources.

Scores

1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

The Marquette University honor code obliges students:

- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.
- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

If you understand and agree to abide by this honor code, sign here:

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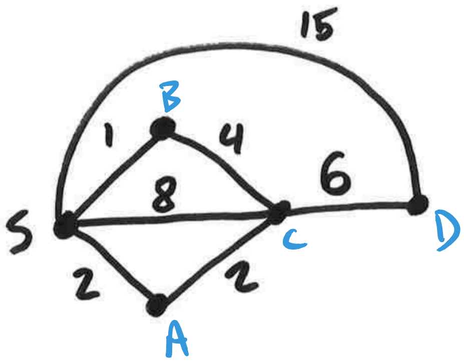
1. Use Dijkstra's Algorithm to compute length of the shortest path from S to any other node. Like the homework, draw a new picture of the graph for each point in the algorithm where you change the "current node". I've given you multiple copies of the graph so you don't have to redraw it every time. (You might use all of them or not, depending on whether you choose to draw one for the start state. That's fine!)

   Make sure you put numbers to indicate the order of your drawings!

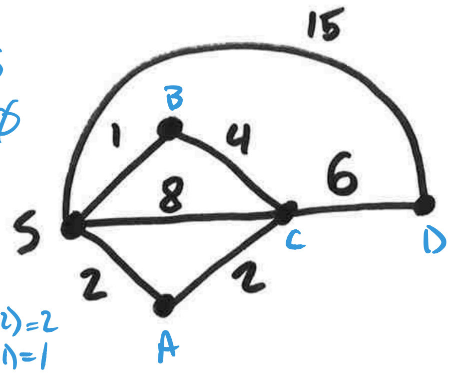
Start (Step 0)



$d_S = 0$
$d_A = \infty$
$d_B = \infty$
$d_C = \infty$
$d_D = \infty$

Step 1

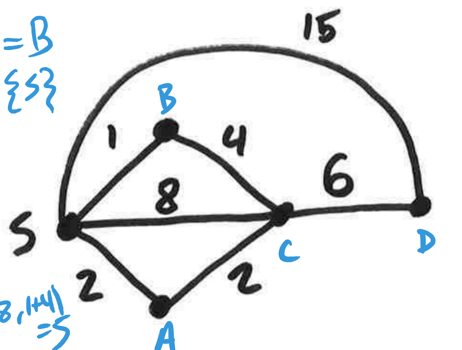
current = S
visited = $\emptyset$



$d_S = 0$
$d_A = \min(\infty, 0+2) = 2$
$d_B = \min(\infty, 0+1) = 1$
$d_C = \min(\infty, 0+8) = 8$
$d_D = \min(\infty, 0+15) = 15$

Step 2

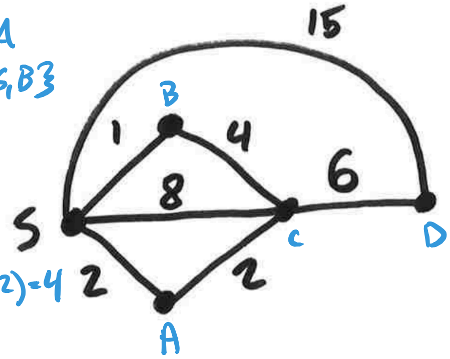
current = B
visited = $\{S\}$



$d_S = 0$
$d_B = 1$
$d_C = \min(8, 1+4) = 5$
$d_A = 2$
$d_D = 15$

Step 3

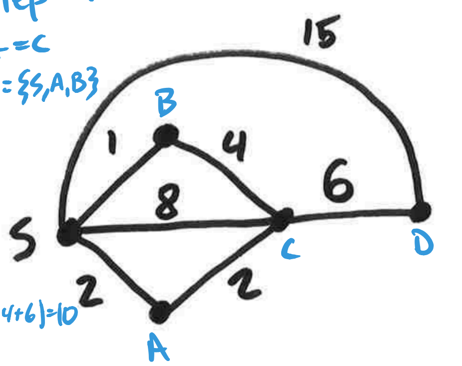
current = A
visited = $\{S, B\}$



$d_S = 0$
$d_A = 2$
$d_B = 1$
$d_C = \min(5, 2+2) = 4$
$d_D = 15$

Step 4

current = C
visited = $\{S, A, B\}$

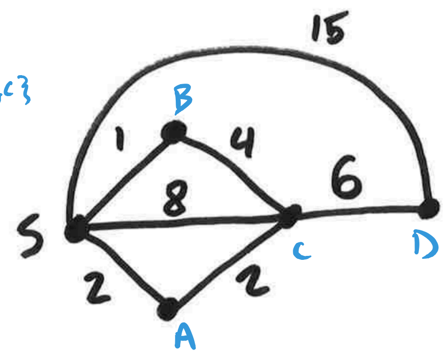


$d_S = 0$
$d_A = 2$
$d_B = 1$
$d_C = 4$
$d_D = \min(15, 4+6) = 10$

Step 5

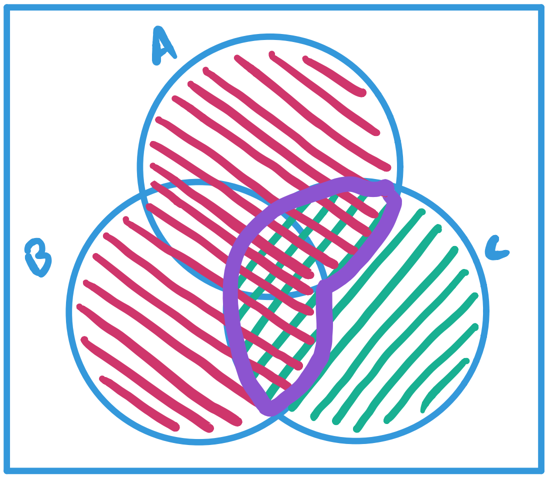
current = D
visited = $\{S, A, B, C\}$

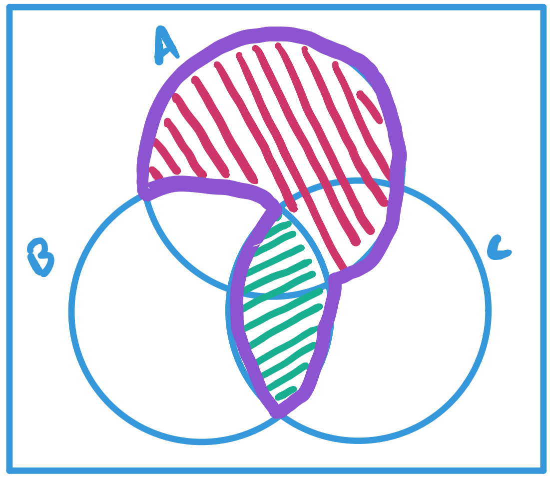
$d_S = 0$
$d_A = 2$
$d_B = 1$
$d_C = 4$
$d_D = 10$



2. Use Venn Diagrams to decide whether the equation below is true.

$$C \cap (A \cup B) = (B \cap C) \cup (A - B).$$





The Venn Diagrams are different, so the sets are <u>not</u> <u>equal</u>.

3. Write the following set T in set-builder notation, and write down five elements in the set:

The set of ordered pairs (a, b) where a is an integer, b is a real number, and b is the square root of a.

$$\{(a,b) \in \mathbb{Z} \times \mathbb{R} : b = \sqrt{a}\}$$

OR

$$\{(a,b) : a \in \mathbb{Z}, b \in \mathbb{R}, b = \sqrt{a}\}$$

5 elements:

$(2, \sqrt{2}), (4, 2), (16, 4), (1, 1), (0, 0)$

4. Use a truth table to determine whether the following logical equivalence is true.

$$(p \wedge \neg q) \vee (r \wedge \neg p) \equiv (r \vee p) \wedge (r \vee \neg q) \wedge (\neg p \vee \neg q)$$

p	q	r	$p \wedge \neg q$	$r \wedge \neg p$		$r \vee p$	$r \vee \neg q$	$\neg p \vee \neg q$	
T	T	T	F	F	F	T	T	F	F
T	T	F	F	F	F	T	F	F	F
T	F	T	T	F	T	T	T	T	T
T	F	F	T	F	T	T	T	T	T
F	T	T	F	T	T	T	T	T	T
F	T	F	F	F	F	F	F	T	F
F	F	T	F	T	T	T	T	T	T
F	F	F	F	F	F	F	T	T	F

These two columns are the same, so the logical equivalence is true.

5. You find three people, Alice, Bob, and Charlie, who say the following:

   Alice: Bob and Charlie are both lying.

   Bob: Exactly two of us are lying.

   Charlie: Alice and Bob are both lying.

   Who is telling the truth and who is lying?

p = Alice is telling the truth
q = Bob is telling the truth
r = Charlie is telling the truth
A = Alice's statement
B = Bob's statement
C = Charlie's statement

p	q	r	A	B	C
T	T	T	F	F	F
T	T	F	F	F	F
T	F	T	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	T	F	F	T	F
F	F	T	F	T	T
F	F	F	T	F	T

This is the only row where $p \equiv A$, $q \equiv B$, and $r \equiv C$,

so,

Alice is lying
Bob is telling the truth
Charlie is lying

6. List five elements in each of the following sets, unless there are fewer than 5 elements in the set (in which case, list all of them and justify how you know you've listed all of the elements).

   (a) $\{x \in \mathbb{R} : x^2 \in \mathbb{N} \text{ and } x \notin \mathbb{N}\}$

   $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}$

   (b) $\{(a,b) \in \{1,2\} \times \{5,7\} : a^2 + b \text{ is odd}\}$

   Since $\{1,2\} \times \{5,7\} = \{(1,5),(1,7),(2,5),(2,7)\}$, this set can't have 5 elements. All the elements are: $\{(2,5),(2,7)\}$

   (c) $\{S \in \mathcal{P}(\{A,B,C,D,E\}) : |S| \text{ is a perfect square}\}$

   Possible elements: any subset of size 0, 1, or 4

   $\{A\}, \{B\}, \{C\}, \{A,C,D,E\}, \{B,C,D,E\}$

7. Consider the implication

$$T = \text{“}\underbrace{\text{If you study in a group}}_{}^{p}\text{, then }\overbrace{\text{you earn 2 tokens}}^{q}\text{.”}$$

(a) What is the converse of T?

$q \to p$

"If you earn two tokens, then you study in a group."

(b) What is the contrapositive of T?

$\neg q \to \neg p$

"If you don't earn two tokens, then you don't study in a group."

(c) What is the inverse of T?

$\neg p \to \neg q$

"If you don't study in a group, then you don't earn two tokens."

(d) Which of the above 3 statements is/are logically equivalent to T?

Contrapositive

8. Consider the following statement.

Every city has a restaurant that makes all of its dishes vegan.

Write the logical negation of this statement. (*Hint:* It will be helpful to first translate to math/quantifiers. [*Double Hint:* There are three quantifiers.])

$P(c,r,d)$ = "dish d at restaurant r in city c is vegan"

C = set of cities
R = set of restaurants
D = set of dishes

Original Statement: $\forall c \in C, \exists r \in R, \forall d \in D, P(c,r,d)$

Negation: $\exists c \in C, \forall r \in R, \exists d \in D, \neg P(c,r,d)$

$\equiv$ "There is a city such that all restaurants in that city have a dish that is not vegan."

9. List all of the elements in the set

$$M = \{(A, B) \in \underbrace{\mathcal{P}(\{1,2\}) \times \mathcal{P}(\{2,3\})}_{\text{ordered pairs of subsets } (S,T), \text{ where}} : |A| \neq |B|\}.$$

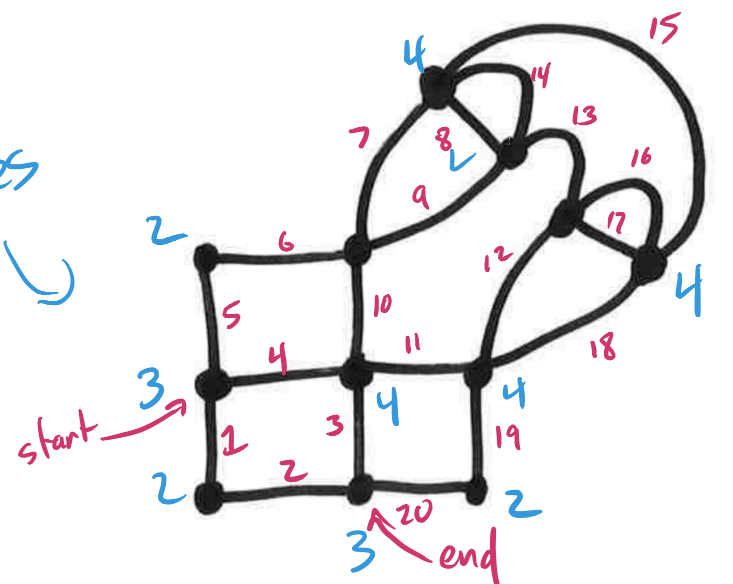
S is one of $\emptyset$ $\{1\}$ $\{2\}$ $\{1,2\}$

T is one of $\emptyset$ $\{2\}$ $\{3\}$ $\{2,3\}$

$$M = \{(\emptyset, \{2\}), (\emptyset, \{3\}), (\emptyset, \{2,3\}),$$
$$(\{1\}, \emptyset), (\{1\}, \{2,3\}),$$
$$(\{2\}, \emptyset), (\{2\}, \{2,3\}),$$
$$(\{1,2\}, \emptyset), (\{1,2\}, \{2\}), (\{1,2\}, \{3\})\}$$

10. For the graph below, find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit. (Note, if they exist, you must actually find one, not just show that one exists.)

degrees



There can only be an Eulerian circuit if there are no vertices of odd degree, so there is no Eulerian circuit.

An Eulerian path must start and end at the two vertices of odd degree. An Eulerian path is shown above with the numbered edges.