MATH 2350 – HOMEWORK 1

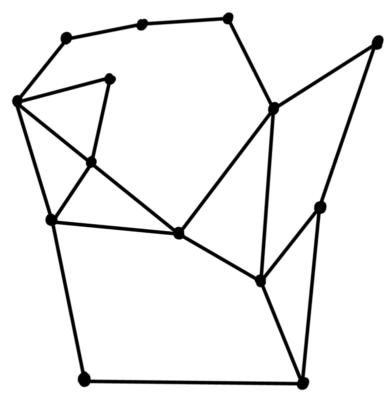
Fall 2025

due Wednesday, **September 17**, at the beginning of class

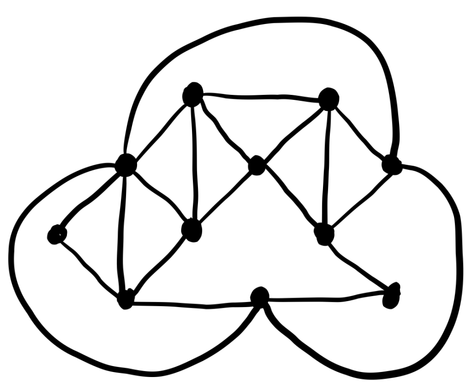
Sections 7.1, 1.3, 1.4

This homework assignment was written in LaTeX. You can find the source code on the course website.

1. For the graph below, label the degree of every vertex. Find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit. *When giving a path or circuit, be sure to find a way to present it on paper that makes sense!*



2. For the graph below, label the degree of every vertex. Find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit. *When giving a path or circuit, be sure to find a way to present it on paper that makes sense!*



3. (a) Draw a graph that *requires* five different colors in order to assign a color to each vertex with the property that no two vertices connected by an edge have the same color.

(b) Draw a graph that *requires* six different colors in order to assign a color to each vertex with the property that no two vertices connected by an edge have the same color.

(c) Explain how for any positive whole number n, you could draw a graph that *requires* n different colors in order to assign a color to each vertex with the property that no two vertices connected by an edge have the same color.

4. In class I stated a theorem that said any map can be colored with at most four colors so that no two regions that border each other have the same color. One of the critical hypotheses of this theorem is that the regions have to be contiguous (they can't have two separate disconnected parts like the state of Michigan's main area and upper peninsula).

   (a) Design a map that is allowed to violate this hypothesis and *requires* at least five colors to draw. This means the "states" can have disconnected parts, but all parts of a state must still have the same color as each other.

   (b) Convert your map to a graph by making the nodes the states and drawing an edge between two states if they share any border.

5. Determine whether the following logical equivalence is true by drawing a truth table. Make sure that you explain how you are using your truth table to draw your conclusion.

$$\neg((r \wedge p) \vee \neg q) \equiv \neg(r \wedge p) \wedge q.$$

6. You come across three inhabitants of an island: Aidan, Disha, and Tareq. Each of them either always lies or always tells the truth. Aidan says "None of us are liars," Disha says "None of us tell the truth," and Tareq says "I'm telling the truth!" What are the possible combinations of whether each person is lying or telling the truth? (There could be no combinations, one combination, or more than one combination!)

7. This is a different type of island-logic-question. You might need another method to answer it. Every inhabitant of an island either always tells the truth or always lies. Each of them knows what type of inhabitant they all are, but you as an outsider do not. You come to a fork in the road and you see two inhabitant there, Alice and Bob. You want to figure out how to get to the airport. The following exchange occurs.

   You: I would like to go the airport.

   Alice: The airport is in the mountains <u>or</u> the road to the right goes to the airport.

   Bob: The airport is in the mountains <u>and</u> the road to the right goes to the airport.

   Alice: Bob is a liar.

   Bob: The road to the right goes to the airport <u>or</u> the airport is not in the mountains.

   Which way is the airport, left or right? (Note: This is as much an exercise in *clearly explaining your logic* as it is in solving the riddle.)

8. Recall that a tautology is a propositional statement that is always true for all possible values of its variables. Which is the following is a tautology?

   (a) $p \wedge (\neg p \vee q)$

   (b) $q \vee r \vee p$

   (c) $p \vee (\neg q \vee p)$

   (d) $(p \wedge q) \vee (\neg p \vee \neg q)$

9. For each of the pairs of statements below, use truth tables to determine whether or not the statements are logically equivalent.

(a) $(p \vee q) \wedge (\neg p \vee q)$ and q

(b) $p \wedge (q \vee r)$ and $(p \wedge q) \vee r$

(c) $(p \vee q) \wedge (q \vee r)$ and $(p \wedge r) \vee q$

10. Let $Q(a, b) =$ "$3a + b = 5a$" and assume for the rest of this question that a and b are always rational numbers (that means they are either 0 or any fraction p/q where p and q are positive or negative whole numbers). Which of the following are true? Justify your answers, stating explicitly whether you're justifying by giving a single example, or by stating something for all cases.

    (a) $Q(1, 2)$

    (b) $\exists x, Q(x, 0)$

    (c) $\exists x, ((x \neq 0) \wedge Q(x, 0))$

    (d) $\forall x, Q(0, x)$

    (e) $\exists y, Q(y, y)$

    (f) $\forall x, \exists y, Q(x, y)$

    (g) $\exists x, \forall y, Q(x, y)$