Math 1450 - Calculus 1

Mon, Dec 1

Announcements:

* HW 13 due Thursday
* Final Exam:
  Wednesday, Dec 10, 8pm-10pm
  Weaster Auditorium

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

→ 5.3: The fundamental theorem

→ 5.4: Theorems about definite integrals

Section 5.3 – The Fundamental Theorem and Interpretations

$\int_a^b v(t)\,dt$ = area under the curve $v(t)$ between $t=a$ and $t=b$

= change in position between $t=a$ and $t=b$

Define $s(t)$ = position at time t

$= s(b) - s(a)$

$$\int_a^b v(t)\,dt = s(b) - s(a)$$

$v(t)$ is the derivative of $s(t)$

This says:

To calculate $\int_a^b v(t)\,dt$:

(1) Find a function $s(t)$ whose derivative is $v(t)$

(2) Do $s(b) - s(a)$

Ex:

$$\int_2^3 2t\,dt$$

Find a function $s(t)$ whose derivative is $2\cdot t$.

$s(t) = t^2$ works

$s(3) - s(2) = 9 - 4 = 5$

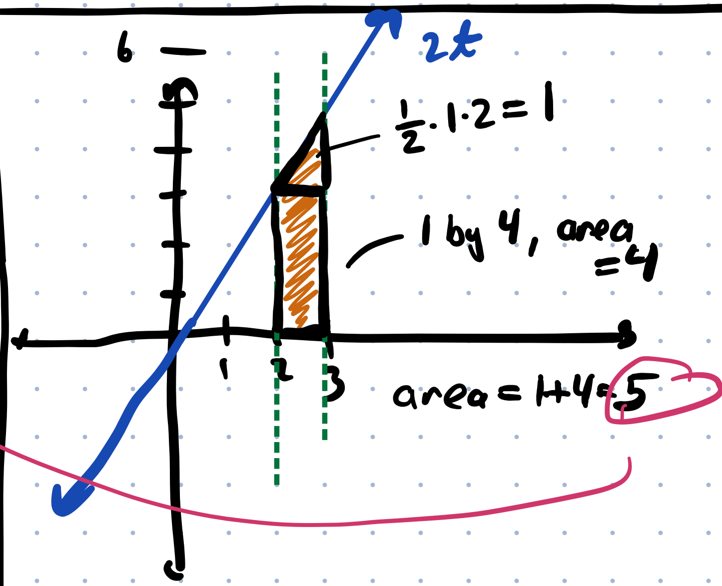
$$\int_{-1}^{4} t^2\,dt \qquad s(t) = \frac{t^3}{3}$$

$s(4) - s(-1)$

$\frac{64}{3} - \left(-\frac{1}{3}\right) = \boxed{\frac{65}{3}}$

If the velocity of a car at time t is $2t$, what is the change in position from $t=2$ to $t=3$?

What is the area under the curve $2\cdot t$ between $t=2$ and $t=3$?



The Fundamental Theorem of Calculus

If f is continuous on the interval $[a,b]$, and if $f(t) = F'(t)$, then

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

Ex: Let $f(t)$ represent a bacterial population that is 5 million at time $t=0$. At time t the population is growing at a rate 2^t million bacteria per hour. What is the population at time $t=1$.

rate of change

$$\int_0^1 2^t \, dt = \text{change in population between } t=0 \text{ and } t=1$$

$$\underbrace{5 \text{ million}}_{\text{start pop.}} + \underbrace{\int_0^1 2^t \, dt}_{\text{amount of change in pop}} = \underbrace{\text{final answer}}_{\text{end pop.}}$$

Ex: Let $f(t)$ represent a bacterial population that is 5 million at time $t=0$. At time t the population is growing at a rate 2^t million bacteria per hour. What is the population at time $t=1$.

$\underbrace{\text{million bacteria per hour.}}_{\text{rate of change}}$

$$\int_0^1 2^t\,dt$$

We need a function $s(t)$ whose derivative is 2^t.

Does 2^t work? $(2^t)' = \ln(2)\cdot 2^t$

Does $\frac{2^t}{\ln(2)}$ work? $\left(\frac{2^t}{\ln(2)}\right)' = \frac{\cancel{\ln(2)}\cdot 2^t}{\cancel{\ln(2)}} = 2^t$. yes

$$\int_0^1 2^t\,dt = \left(\frac{2^1}{\ln(2)}\right) - \left(\frac{2^0}{\ln(2)}\right) = \frac{2}{\ln(2)} - \frac{1}{\ln(2)} = \frac{1}{\ln(2)} \approx 1.44$$

Ex: Let $f(t)$ represent a bacterial population that is 5 million at time $t=0$. At time t the population is growing at a rate 2^t million bacteria per hour. What is the population at time $t=1$.

rate of change

Answer: $5 + \int_0^1 2^t \, dt \approx 5 + 1.44$

$$\boxed{\approx 6.44 \text{ million bacteria}}$$

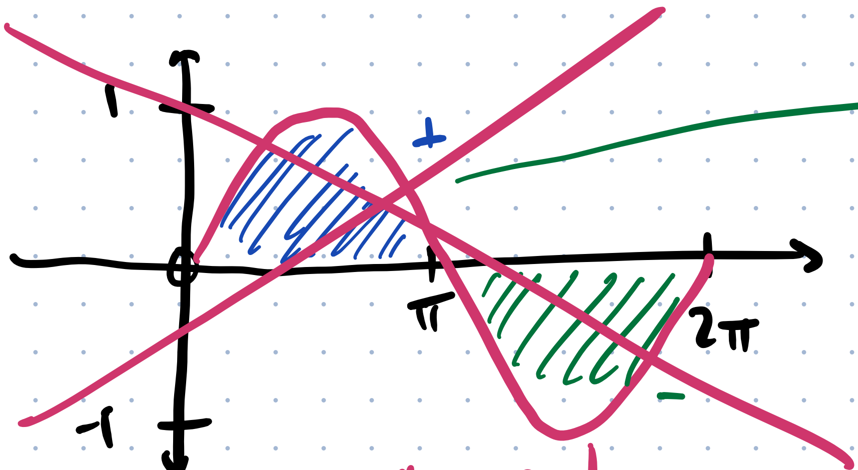
Ex: $\int_0^{2\pi} \cos(\theta)\, d\theta$

A function whose derivative is $\cos(\theta)$:

$\sin(\theta)$

$= \sin(2\pi) - \sin(0)$

$= 0 - 0 = 0$



wrong graph

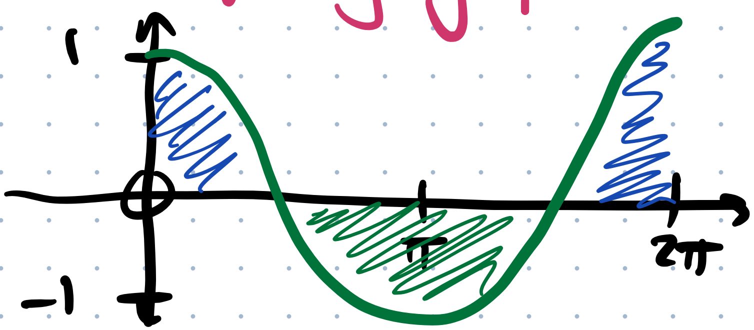
Blue area:

$$\int_0^1 \underbrace{\sin(\theta)}_{f}\, d\theta$$

$F = -\cos(\theta)$

$-\cos(\pi) - (-\cos(0))$

$1 + 1 = 2$



Ex: $\displaystyle\int_2^5 \frac{2}{x}\,dx$

look for a function whose derivative is $\frac{2}{x}$

$=$

$\boxed{2\cdot\ln(5)-2\cdot\ln(2)}$

the derivative of $\ln(x)$ is $\frac{1}{x}$

the derivative of $2\cdot\ln(x)$ is $\frac{2}{x}$

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the derivative of $2\cdot\ln(x)+17$ is also $\frac{2}{x}$

$\left(2\cdot\ln(5)\cancel{+17}\right)-\left(2\cdot\ln(2)\cancel{+17}\right)$

$\boxed{2\cdot\ln(x)+C}$

Section 5.4: Theorems about Definite Integrals

Fact #1: If f is continuous and a and b are any #s:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Ex: If $\int_3^5 f(x)\,dx = 10$, then $\int_5^3 f(x)\,dx = -10$.

Fact #2: If f is continuous and a, b, c are any #s, then:

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

orange area + green area = purple area