# Math 1450 - Calculus 1

Mon, Nov 24

## Announcements:

* This Week: lecture today
  discussion Tuesday
  nothing Wed, Thurs, Fri
* HW 13 due Thurs, Dec 4 - start now!
* Final Exam:
  Wednesday, Dec 10, 8pm-10pm
  Weaster Auditorium

**Office Hours**

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

## Today:

→ 5.2: The definite integral

→ 5.3: The fundamental theorem

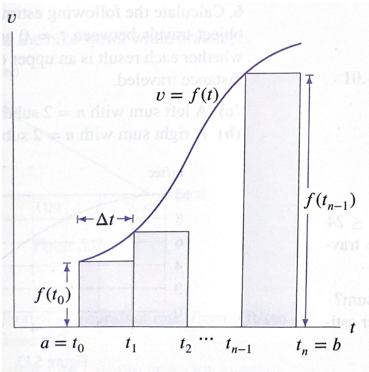


Figure 5.8: Left-hand sums

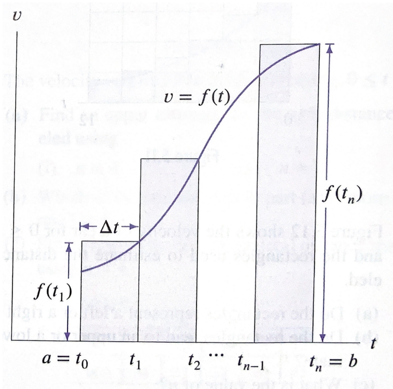
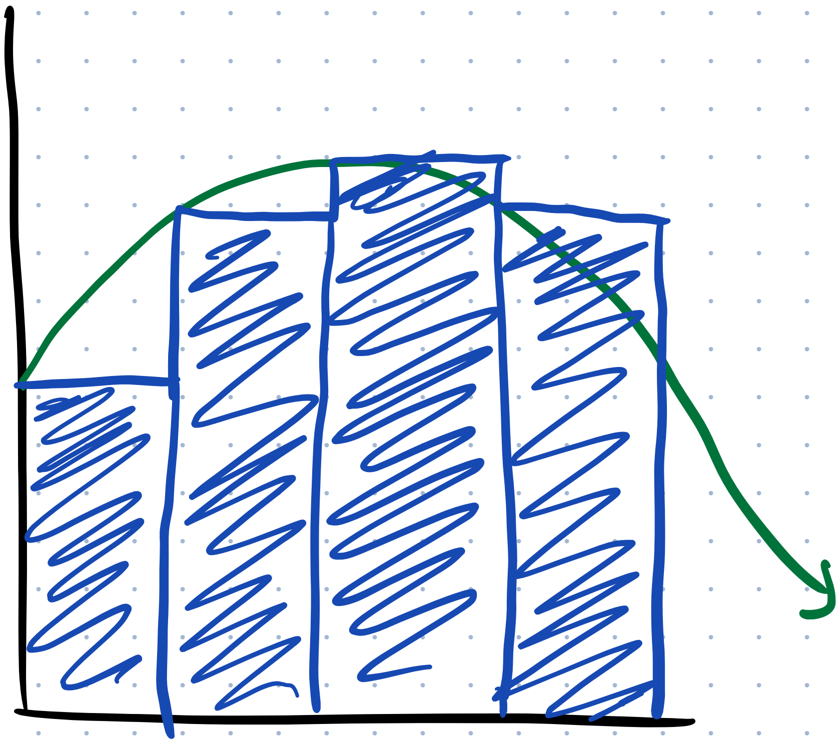


Figure 5.9: Right-hand sums

$0 \le t \le 12$



# Section 5.2: The Definite Integral

opposite of a derivative

"$\Sigma$" capital greek sigma $\sigma$

Let "P(i)" be some mathematical expression in terms of the variable $i$.

$$\sum_{i=a}^{i=b} P(i) = P(a) + P(a+1) + P(a+2) + \dots + P(b-1) + P(b)$$

"add P(i) for all values of i (whole #s) between a and b"

$$\sum_{i=3}^{6} i^2 = 3^2 + 4^2 + 5^2 + 6^2 = 86$$

$$\text{Left} = \Delta t \cdot \left( f(t_0) + f(t_1) + \dots + f(t_{n-1}) \right)$$

$$= \sum_{i=0}^{n-1} (\Delta t) \cdot f(t_i)$$

$$= (\Delta t) \cdot \sum_{i=0}^{n-1} f(t_i)$$

$$\text{Right} = \Delta t \cdot \left( f(t_1) + f(t_2) + \dots + f(t_n) \right)$$

$$= \sum_{i=1}^{n} (\Delta t) \cdot f(t_i)$$

$$= (\Delta t) \sum_{i=1}^{n} f(t_i)$$

"Riemann sums" – the name for these left and right sums

As the number of rectangles (n) gets bigger and bigger, the approximation to the true area under the curve gets better and better.

So, we're going to take the limit as $n \to \infty$

# The Definite Integral

We use the notation "$\displaystyle\int_a^b f(t)\,dt$"

integral sign, fancy "S" for "sum"

to mean:

the (signed) area under the curve $f(t)$ between $t=a$ and $t=b$.

above the axis counts as positive
below the axis counts as negative

$$\int_a^b \overbrace{f(t)}^{\text{the function}}\,dt$$

$b$: right endpoint

$a$: left endpoint

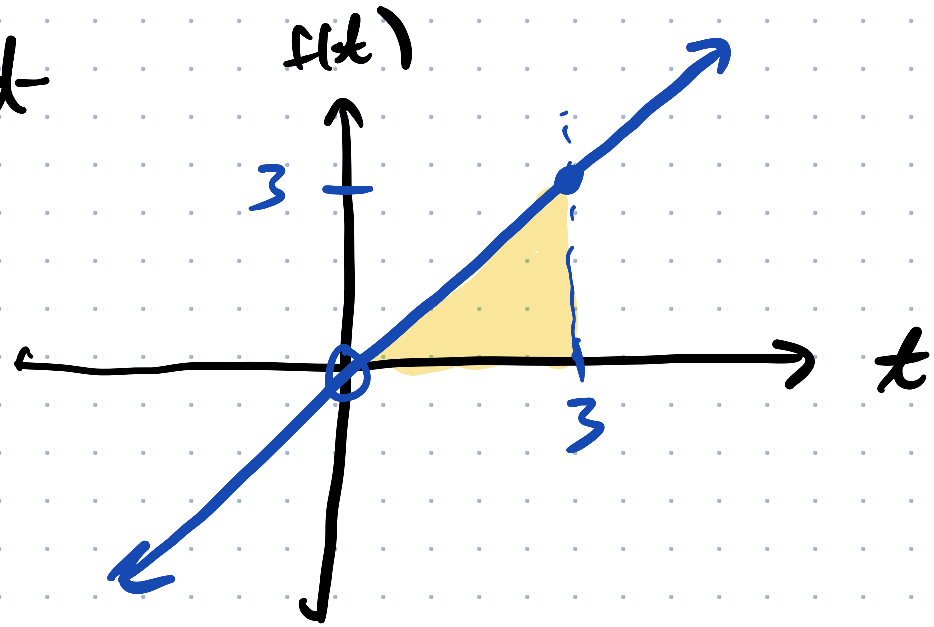
$dt$ tells us the variable

just like $\frac{d}{dx}$ tells us the variable

## Ex:

$$\int_0^3 t \, dt$$

$f(t) = t$




$A = \frac{1}{2} \cdot b \cdot h$

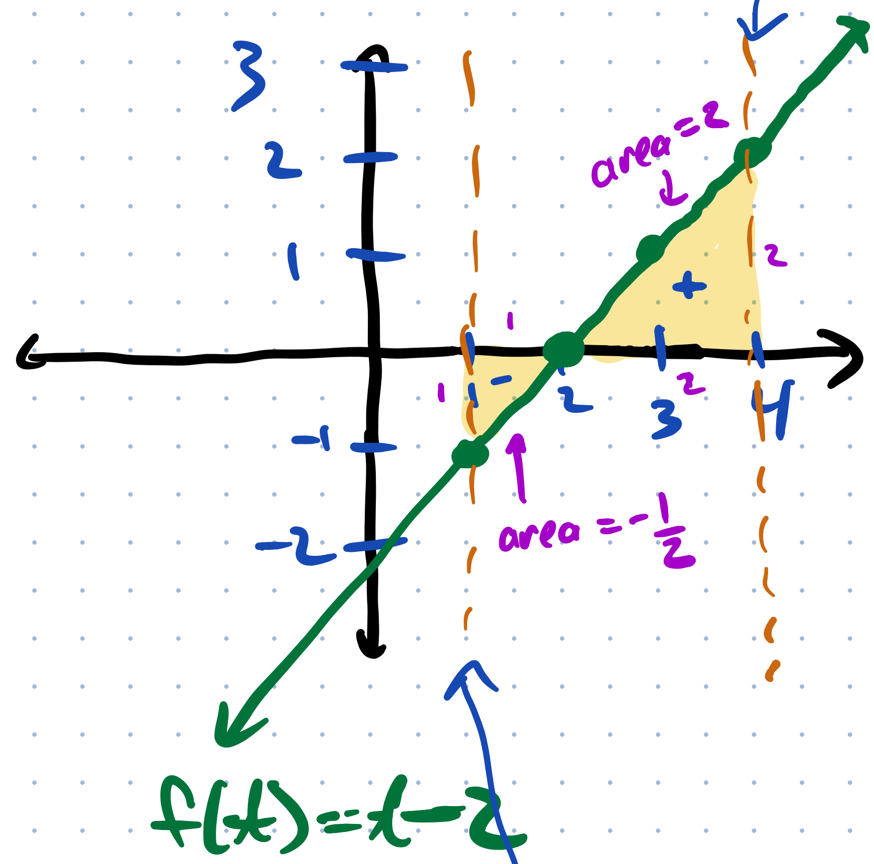
$A = \frac{1}{2} \cdot 3 \cdot 3$

$\boxed{= 4.5}$

$$\int_0^3 t \, dt = 4.5$$

Ex:

$$\int_{1}^{4} (t-2)\,dt = 1.5 = \frac{3}{2}$$



## Facts

* $\int_a^b f(t)\,dt$ is approximated by adding the area of left or right Riemann sums

* More rectangles = better approximation

So:

$$\int_a^b f(t)\,dt = \lim_{n\to\infty} (\text{LH sum})$$

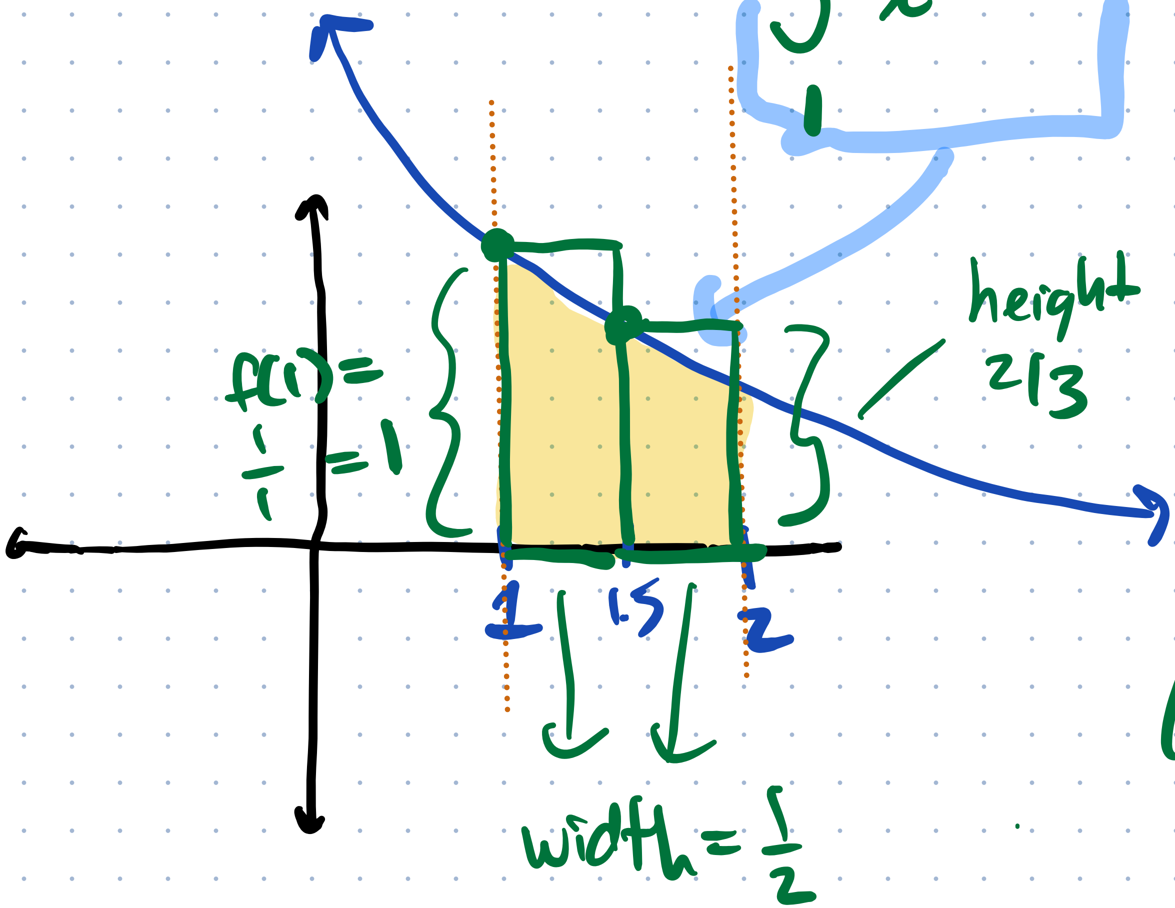
$$= \lim_{n\to\infty} \left( \sum_{i=0}^{n-1} f(t_i)\cdot \Delta t \right)$$

RH sum version:

$$\int_a^b f(t)\,dt = \lim_{n\to\infty}\left(\text{RH sum}\right)$$

$$= \lim_{n\to\infty}\left(\sum_{i=1}^{n} f(t_i)\cdot\Delta t\right)$$

Ex: Estimate $\int_1^2 \frac{1}{t}\,dt$ with a LH sum and $n=2$.




$$f(1.5) = f\left(\tfrac{3}{2}\right) = \frac{1}{3/2} = \frac{2}{3}$$

$$\left(\tfrac{1}{2}\right)(1) + \left(\tfrac{1}{2}\right)\cdot\left(\tfrac{2}{3}\right) = \frac{1}{2} + \frac{1}{3} = \boxed{\frac{5}{6}} = 0.833\ldots$$

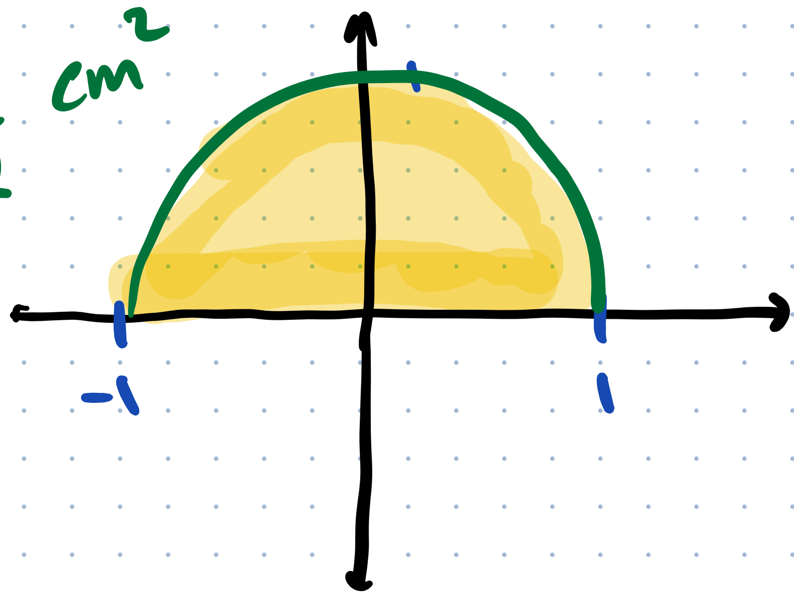
actual area $= \ln(2) \approx 0.693$

Ex: The top half of a circle with radius 1 cm is described by the function $y = \sqrt{1-x^2}$.

What is $\int_{-1}^{1} \sqrt{1-x^2} \cdot dx$?

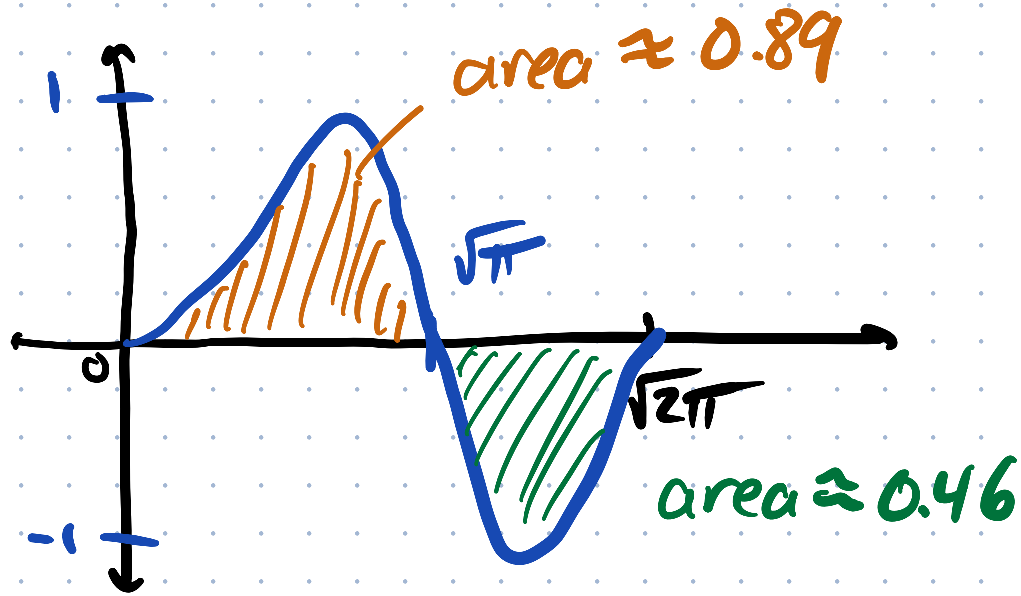
area of a full circle with radius 1 is $\pi \cdot (1)^2 = \pi \text{ cm}^2$.

$$\int_{-1}^{1} \sqrt{1-x^2}\, dx = \frac{\pi}{2} \text{ cm}^2$$



Ex: (area below the axis counts as negative)

$$\int_0^{\sqrt{2\pi}} \sin(\theta^2)\,d\theta$$




$$\approx 0.89 + (-0.46)$$

$$\boxed{\approx 0.43}$$

# Section 5.3 – The Fundamental Theorem and Interpretations

$\int_a^b v(t)\,dt$ = area under the curve $v(t)$ between $t=a$ and $t=b$

= change in position between $t=a$ and $t=b$

Define $s(t)$ = position at time $t$

$= s(b) - s(a)$

$$\int_a^b v(t)\,dt = s(b) - s(a)$$

$v(t)$ is the derivative of $s(t)$

This says:

To calculate $\int_a^b v(t)\,dt$:

(1) Find a function $s(t)$ whose derivative is $v(t)$

(2) Do $s(b) - s(a)$