Math 1450 - Calculus 1

Mon, Nov. 3

Announcements:

* HW 10 due on Thursday, Nov. 6 - covers 4.1 and 4.2
* Quiz 8 on Thursday, covers all 4.1 + 4.2 sugg. HW
* Exam 3 on Wednesday, Nov. 12
  covers 3.5, 3.6, 3.7, 3.9, 3.10
  4.1, 4.2, 4.3, 4.6

Office Hours
Mondays, 12-1
Wednesdays, 2-3
+ Help Desk! 12-1

Today:

→ 4.2: Optimization
→ 4.3: Optimization + Modeling

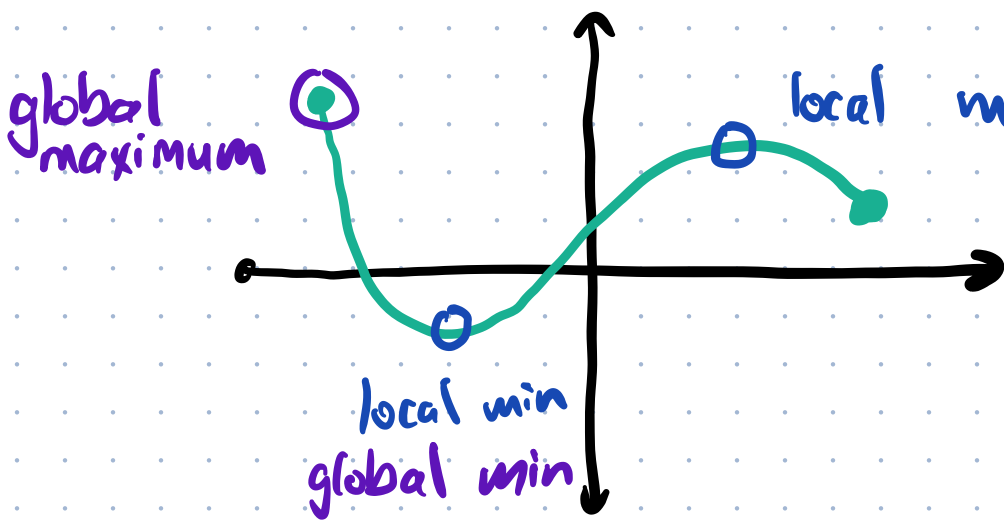
4.2: Optimization

4.1: Goal: Find all local minima and local maxima, and inflection points.

<u>Optimization</u>: Finding where a function is largest or smallest anywhere.

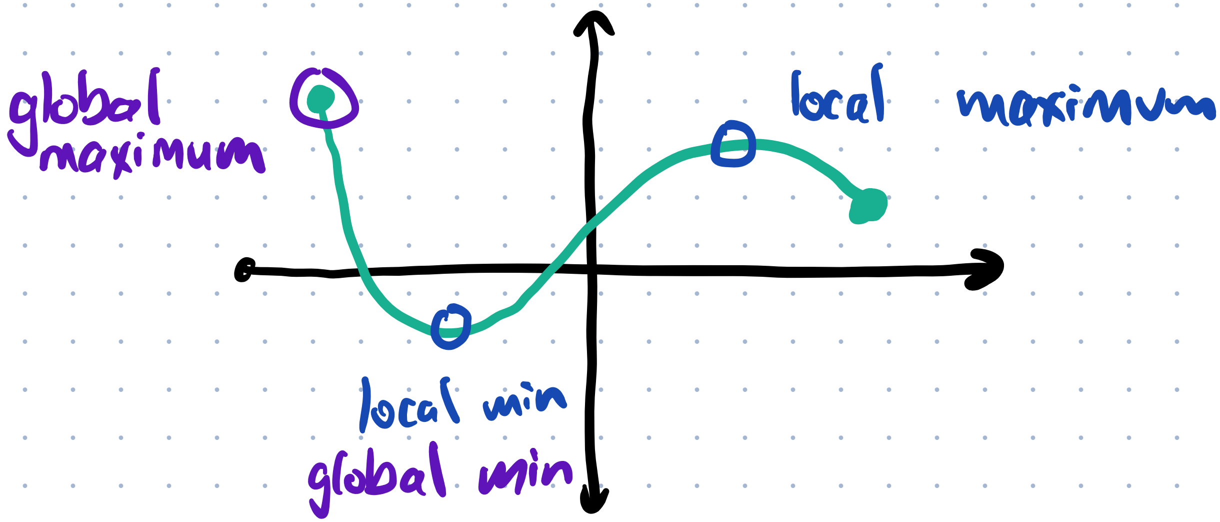
Global minimum: where is a function smallest <u>ever</u>

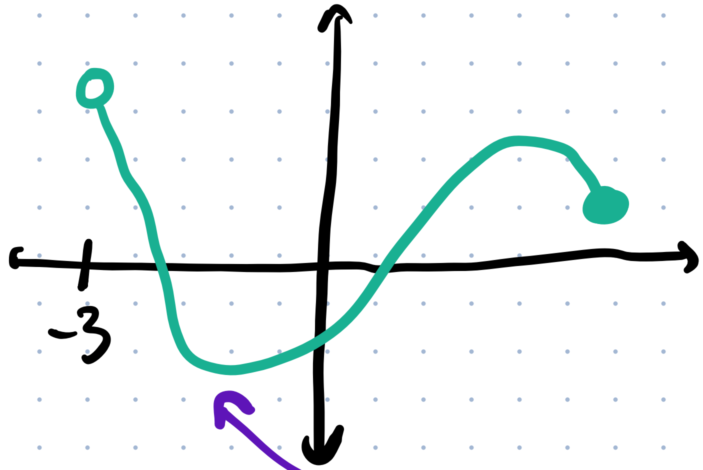
Global maximum: where is it largest ever



Fact: If f is continuous on a closed interval $[a,b]$ (has endpoints), then f must have both a global max and a global min somewhere in $[a,b]$.

example



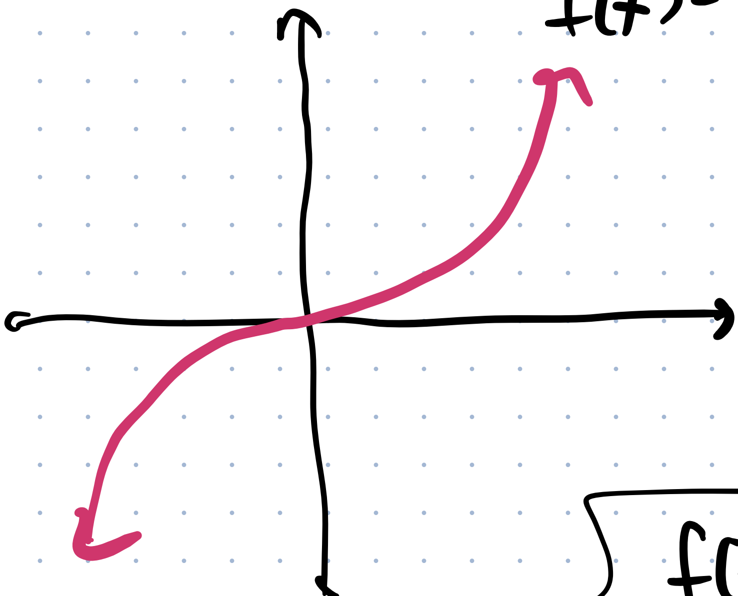


no global maximum

yes global minimum

there is no single x-val. that has the largest y-value

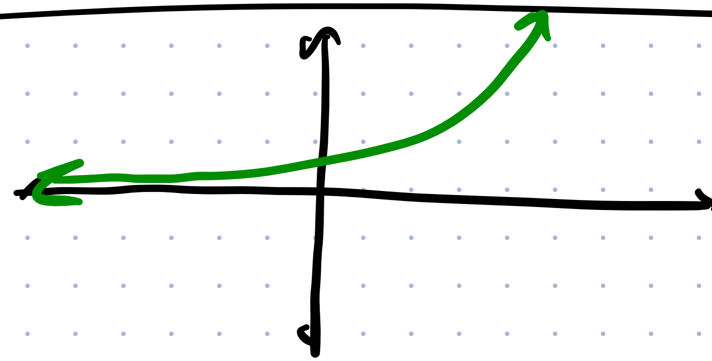
$f(x)=x^3$



no local extrema

no global extrema

$f(x)=e^x$



no local extrema

no global extrema

Steps

* Find the critical points.

* Plug the critical points and the endpoints if they exist back into the function to find their y-values

Biggest = maybe the global max
Smallest = maybe the global min

if both endpoints exist, then the "maybe" becomes "definitely."

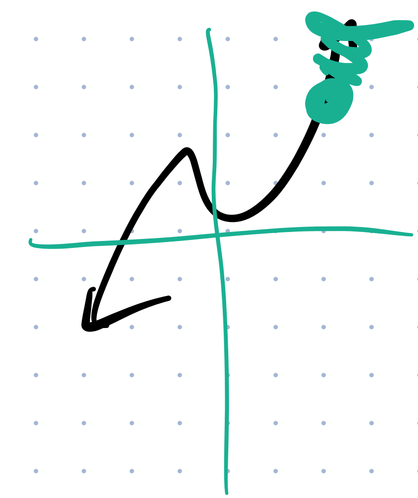
Every global max or min occurs at either a local max or min or an endpoint

If we're missing endpoints, we have to think about how the graph looks.

Ex: Find the global extrema of

$$f(x) = x^3 - 9x^2 - 48x + 52$$

on the interval $(-\infty, 14]$



$$-\infty < x \leq 14$$

$$3x^2 - 18x - 48 = 3\cdot(x+2)(x-8)$$

Step 1: Critical points:

In 4.1, we did this example:

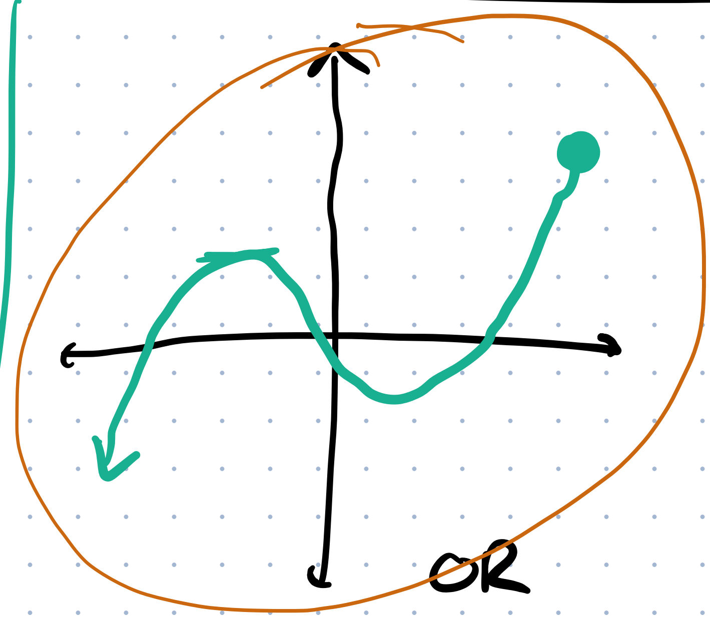
$x = -2,\ 8$

endpoints $x = 14$

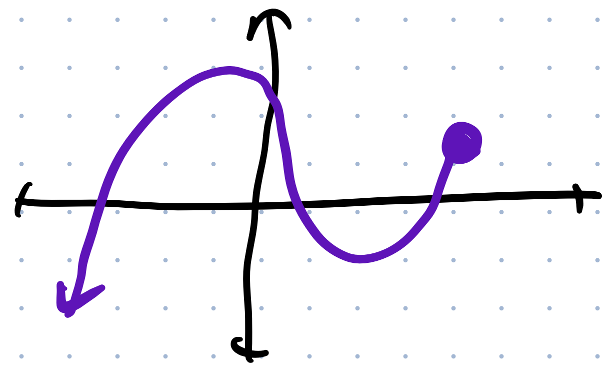
$$f(-2) = (-2)^3 - 9\cdot(-2)^2 - 48(-2) + 52 = 104$$

$f(8) = -396$

$f(14) = 360$



OR



global maximum: $(14, 360)$

global minimum: $\cancel{(8, -396)}$ DNE

no global minimum

Takes lots of practice.

4.3: Optimization and Modeling

This section: "Optimize some quantity, subject to some constraint"

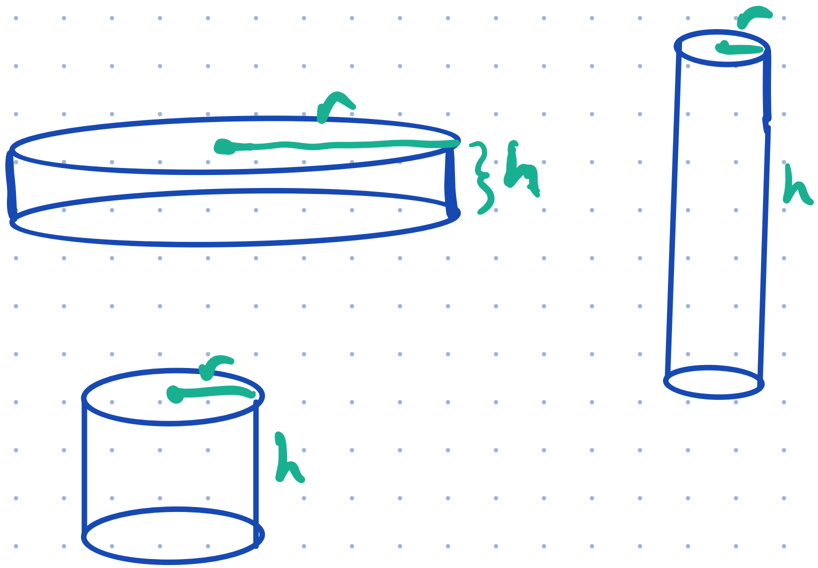
Find global max or global min

volume of a box	fixed surface area
surface area of a box	fixed volume
area of a fenced in yard	fixed amount of fence
gas consumption	fixed distance

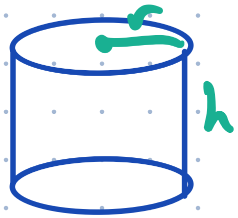
Ex: (from book) What are the dimensions of an aluminum can (cylinder) that holds 40 in^3 of juice and uses the least amount of material?

volume constraint

surface area
quantity we want to minimize



$\boxed{\begin{array}{l}\text{Constraint: Volume} = 40 \\ \text{Goal: minimize surface area}\end{array}}$



Volume = $\boxed{\pi r^2 \cdot h}$ area of a circle $= \pi r^2$

Surface area = $\underbrace{2\pi r^2}_{\text{top + bottom}} + \underbrace{h \cdot (2\pi r)}_{\text{outside}}$

Two variables, r and h

The constraint tells us how r and h must be related.

$\boxed{\pi r^2 h = 40}$

* Use the constraint to solve for one variable in terms of the other.

solve for h

$h = \dfrac{40}{\pi r^2}$

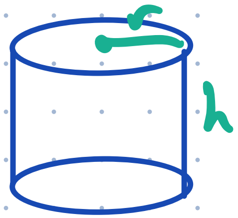
solve for r

$r = \sqrt{\dfrac{40}{\pi h}}$

pick the one that looks nicest

Constraint: $\boxed{\text{Volume} = 40}$

Goal: minimize surface area



Volume = $\boxed{\pi r^2 \cdot h}$

Surface area = $\underbrace{2\pi r^2}_{\text{top+bottom}} + \underbrace{h\cdot(2\pi r)}_{\text{outside}}$

$h = \frac{40}{\pi r^2}$

plug in $h = \frac{40}{\pi r^2}$ so we only have one variable

Surface area:

$$2\pi r^2 + 2\pi r \cdot h = 2\pi r^2 + 2\cancel{\pi r}\cdot\left(\frac{40}{\cancel{\pi r^2}}\right)$$

$$\boxed{S(r) = 2\pi r^2 + \frac{80}{r}}$$

Use the stuff from 4-2 to find the global minimum of S(r).