Math 1450 – Calculus 1

Fri, Oct. 31

Announcements:

Happy Halloween!

* HW 10 due on Thursday, Nov. 6 – covers 4.1 and 4.2
* Quiz 8 on Thursday, covers all 4.1 + 4.2 sugg. HW
* Exam 3 on Wednesday, Nov. 12
  covers 3.5, 3.6, 3.7, 3.9, 3.10
  4.1, 4.2, 4.3, 4.6

Today:

→ 4.1: Using First and Second Derivatives
→ 4.2: Optimization

Office Hours

Mondays, 12-1

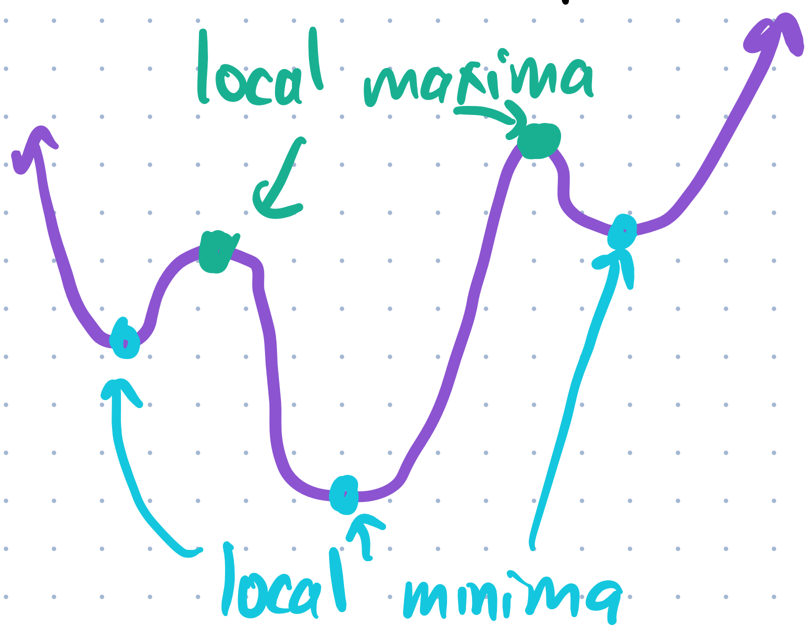
Wednesdays, 2-3

+ Help Desk! 12-1

Local Maxima and Minima (local vs. global)

* A local minimum is a point of a function whose value is smaller than all of the points near it. (a valley)

* A local maximum is a point of a function whose value is larger than all of the points near it. (a peak)



"maxima" is the plural of maximum

same for minima

Finding local minima and maxima:

Idea: When there's a local max or min, the derivative must be 0 or undefined.

Definition:

A **critical point** is an x-value $x=p$ in the domain of $f(x)$ where either:

* $f'(p) = 0$

OR

* $f'(p)$ is undefined

maxima or minima?

Theorem: All local extrema occur at critical points.

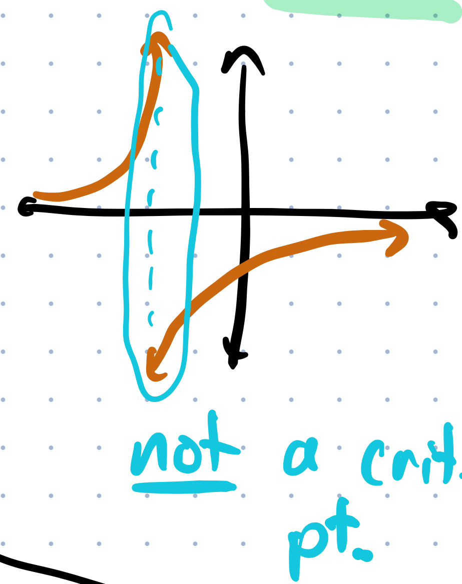
Warning: not all critical points are local extrema!!

Algorithm for finding local extrema of f(x).

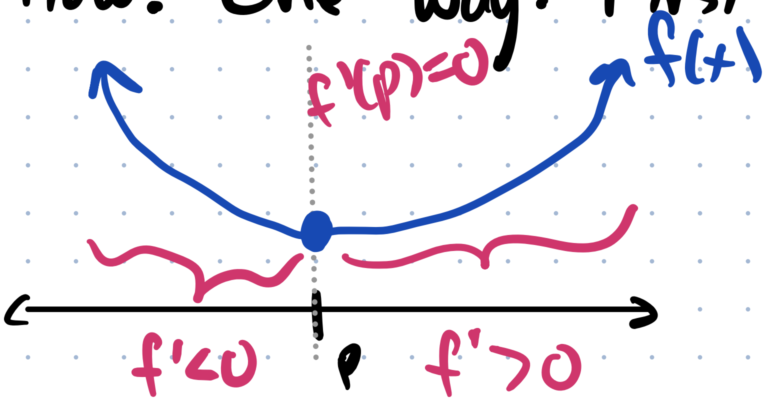
(1) Find the critical points (candidates)

- compute $f'(x)$
- solve for where $f'(x)=0$
- also include where f' is undefined but f is defined

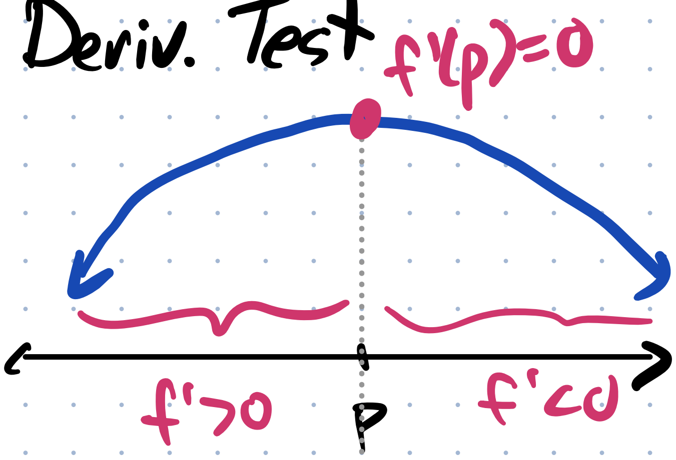
(2) Check each critical point to see if it's a local min, local max, or neither.



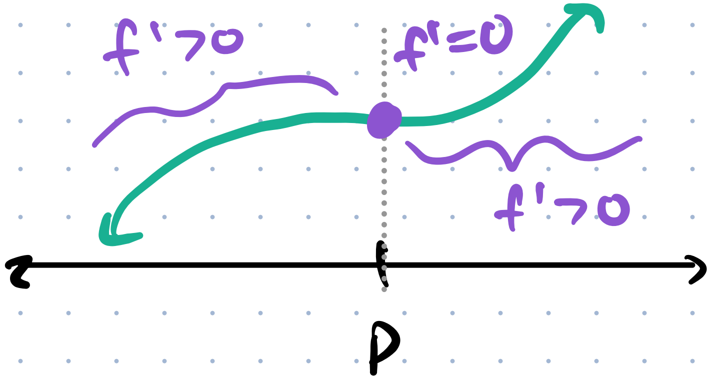
How? One way: First Deriv. Test



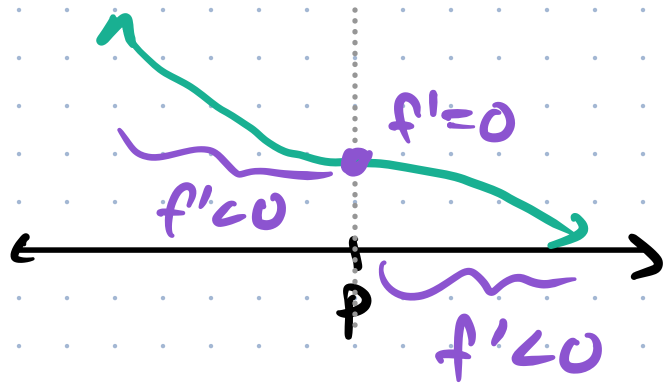
local minimum



local maximum



neither local max nor a local min



neither

How will you determine whether f' is $+/-$ on either side?

The whole number line thing

Ex: $f(x) = \dfrac{1}{x(x-1)}$

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$f'(x) = -\dfrac{2x-1}{(x^2-x)^2}$ $= 0$? undefined?

$2x-1=0 \Rightarrow x = \frac{1}{2}$

$(x^2-x)^2 = 0$

$\Rightarrow x^2 - x = 0$

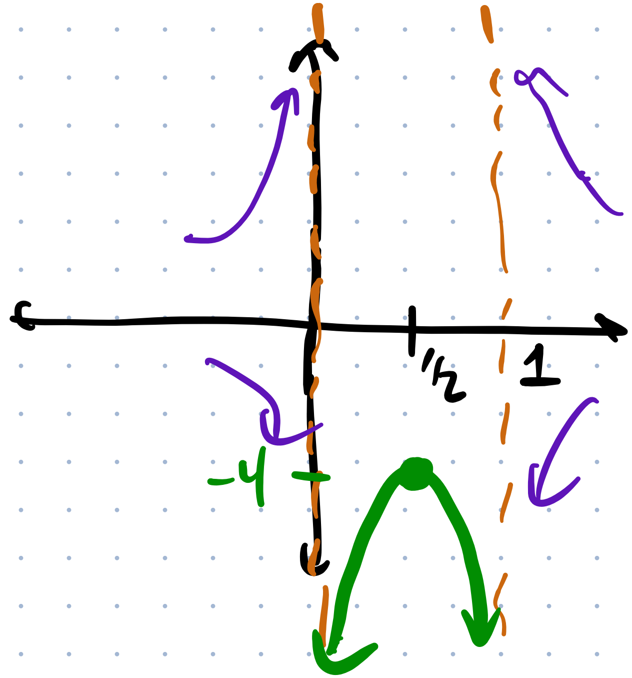
$\Rightarrow x(x-1) = 0$

$\Rightarrow x=0$ or $x=1$

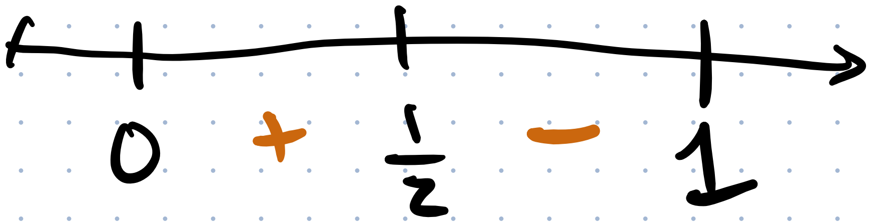
not critical points because 0 and 1 are not in the domain of f (f is undefined)

Only critical point: $\frac{1}{2}$

Ex: $f(x) = \dfrac{1}{x(x-1)}$



$$f'(x) = -\frac{2x-1}{(x^2-x)^2} = -\frac{2x-1}{(x(x-1))^2}$$



$$f'\left(\tfrac{1}{4}\right) = -\frac{\left(2\cdot\frac{1}{4}-1\right)}{\left(\left(\frac{1}{4}\right)\left(\frac{1}{4}-1\right)\right)^2}$$

$$= - \cdot \frac{-}{+} = +$$

$$f'\left(\tfrac{3}{4}\right) = -\frac{\left(2\cdot\frac{3}{4}-1\right)}{+} = -\frac{+}{+} = -$$

First Derivative Test

$$f\left(\tfrac{1}{2}\right) = \frac{1}{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)} = -4$$

Second Derivative Test (Alternative to the F.D.T.)

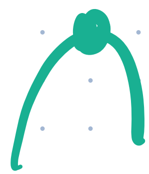
Still find the crit. pts. the same way

Then check $f''(p)$ at the critical pt.

For any critical point p, three cases:

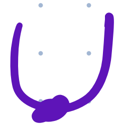
(1) If $f''(p) > 0$ then p is a local minimum.

(2) If $f''(p) < 0$ then p is a local maximum.



(3) If $f''(p) = 0$, then we can draw no conclusion.

x^3 $\quad$ $6x$



x^4 $\quad$ $12x^2$

Earlier example $\quad f(x) = x^3 - 9x^2 - 48x + 52$

$$f'(x) = 3x^2 - 18x - 48$$

$$f''(x) = 6x - 18$$

critical points: $p = -2$, $\quad p = 8$

$$f''(-2) = 6(-2) - 18 = -12 - 18 = \boxed{-30}$$

f is cc$\downarrow$ at $x = -2$ $\quad \cap$, $\quad -2$ is a local max

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$$f''(8) = 6 \cdot 8 - 18 = \boxed{30}$$

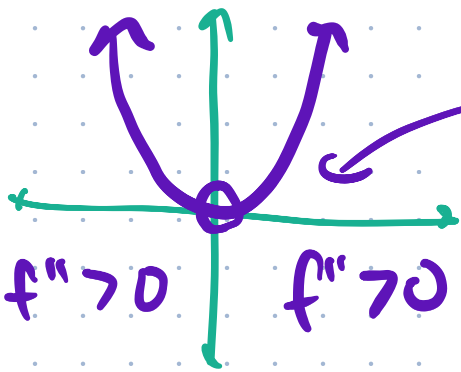
f is cc$\uparrow$ at $x = 8$ $\quad \cup$, $\quad x = 8$ is a local min

Definition: In addition to local extrema (f' changes from $+$ to $-$ or $-$ to $+$), we can also find "inflection points", where f'' changes from $+$ to $-$ or $-$ to $+$.

To inflection points, find where $f''=0$ or f'' is undefined, then check if f'' changes sign from the left to the right.

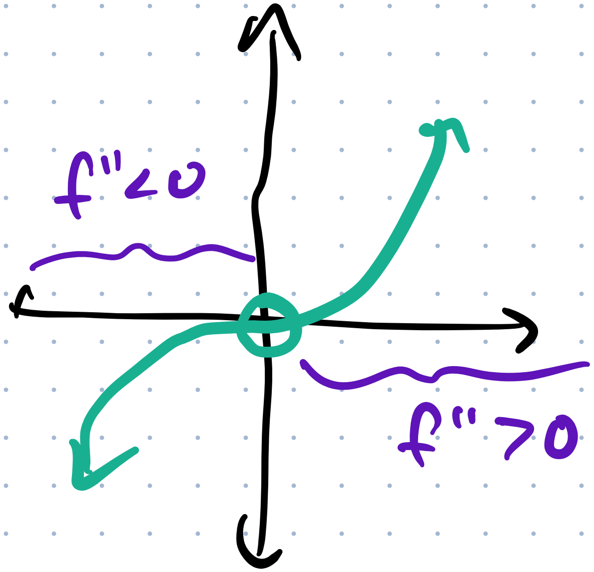
Ex: $f(x)=x^4$:

$f''=0$ at $x=0$



not really an infl. pt. because f'' does not change signs

Ex: $f(x) = x^3$



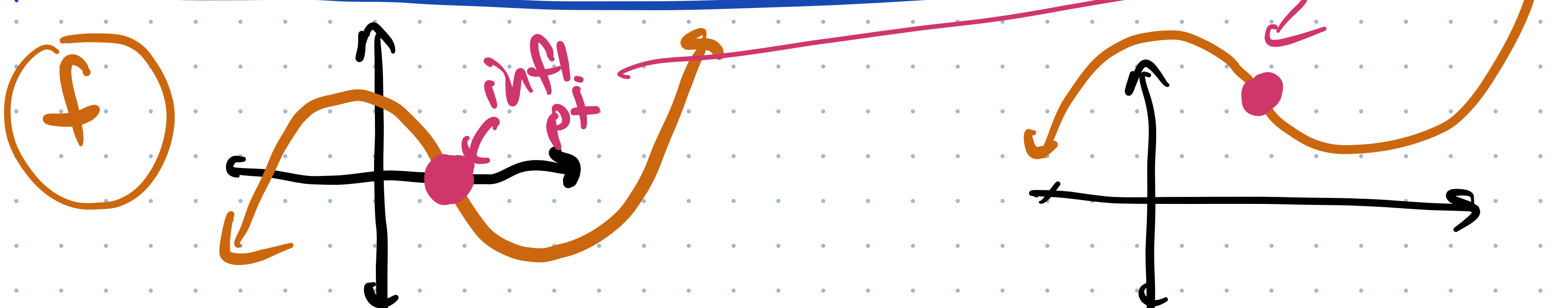
$f''(x) = 6x$

$x = 0$

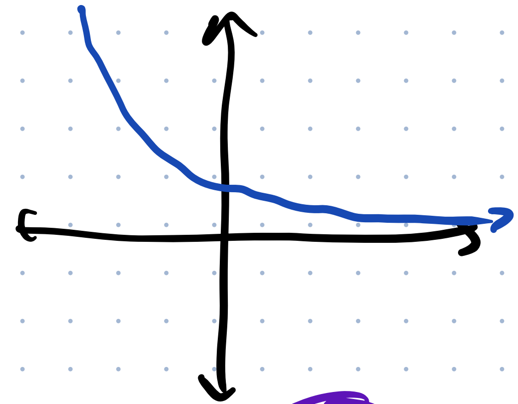
$f''(-1) = -6$ $\ominus$

$f''(1) = 6$ $\oplus$

$x=0$ <u>is</u> an inflection point.



Ex: Find all local extrema and all inflection points of $g(x)=x\cdot e^{-x}$, then graph as much of it as you can.



$g'(x)=e^{-x}-xe^{-x}=e^{-x}\cdot(1-x)$

$g''(x)=(x-2)\cdot e^{-x}$

critical points: $x=1$, $g''(1)=-\cdot+=-$

local max

$g''(x)=0 \Rightarrow \boxed{x=2}$ infl. pt.

$g''(1)=(1-2)\cdot e^{-1}=-\cdot+=-$

$g''(3)=(3-2)\cdot e^{-3}=+\cdot+=+$