Math 1450 - Calculus 1

Fri, Sept. 12

Announcements:

* HW 3 due Tuesday, 11:59pm
  covers 1.7 - 1.9

* Exam 1 - Wednesday, Sept. 17, 5pm-6pm, this room
  * study guide on course website!
  * covers 1.1-1.9
  * calculators allowed (nothing with wifi/bluetooth)

Today:

→ 1.8: Extending the Idea of a Limit

→ 1.9: Further Limit Calculations Using Algebra

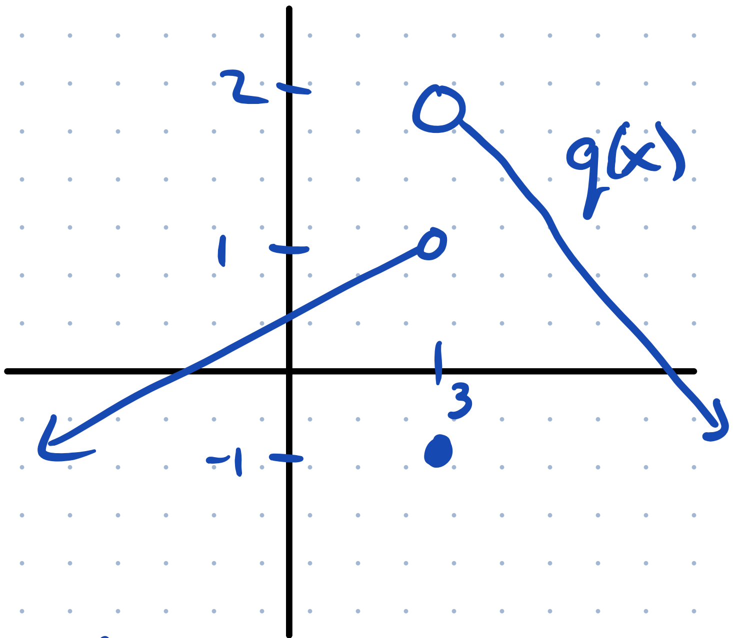
Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Ex:

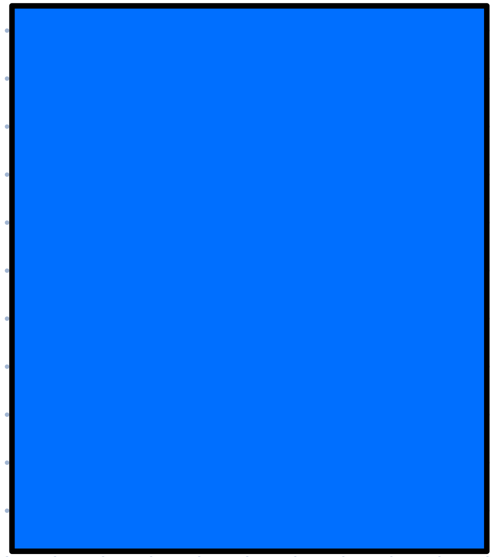


$g(3) = -1$

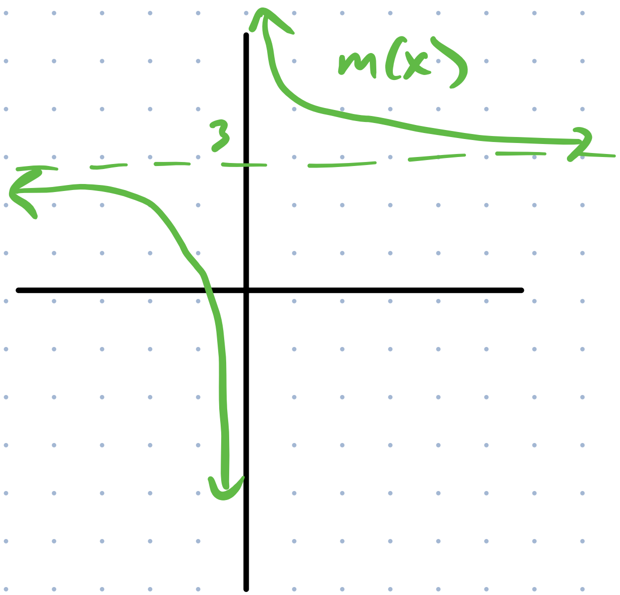
$\lim_{x \to 3^+} g(x) = 2$

$\lim_{x \to 3^-} g(x) = 1$

$\lim_{x \to 3} g(x) = \text{DNE}$



Limits can help us describe horizontal and vertical asymptotes too.



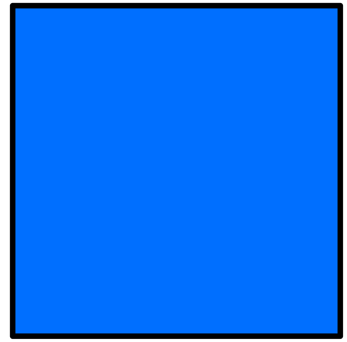
$$\lim_{x \to \infty} m(x) = 3$$

$$\lim_{x \to -\infty} m(x) = 3$$

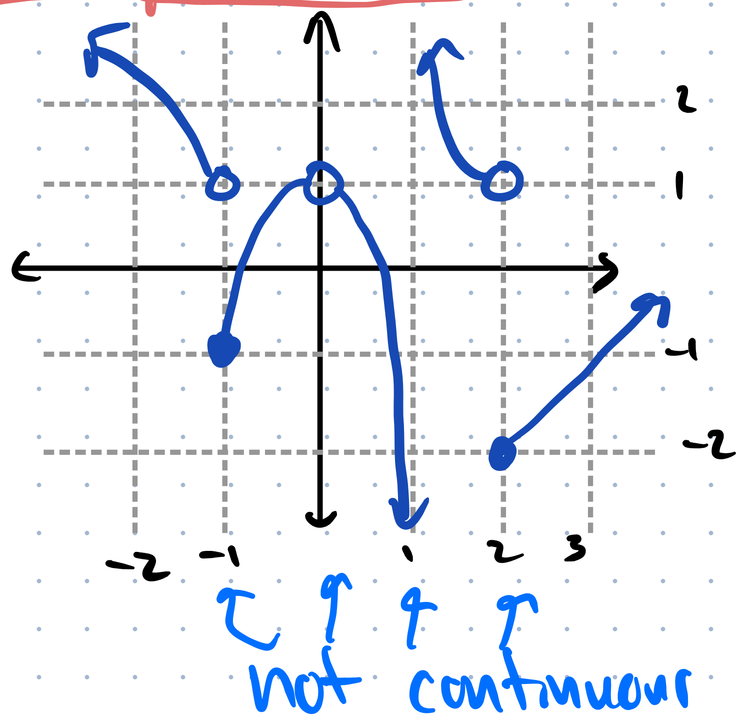
$$\lim_{x \to 0} m(x) = \text{DNE}$$

$$\lim_{x \to 0^+} m(x) = +\infty$$

$$\lim_{x \to 0^-} m(x) = -\infty$$



Group Work:



0 is not in the domain

Find $f(-1)$, $f(0)$, $f(1)$, $f(2)$ and both one-sided limits and the two-sided limits at $x = -1, 0, 1, 2$.

On what intervals is this function continuous? ~~$(-\infty, 1]$~~

a	-1	0	1	2	3
$f(a)$	-1	DNE	DNE	-2	-1
$\lim_{x\to a^-} f(x)$	1	1	$-\infty$	1	-1
$\lim_{x\to a^+} f(x)$	-1	1	$+\infty$	-2	-1
$\lim_{x\to a} f(x)$	DNE	1	DNE	DNE	-1

$(-\infty, -1)$

$(-1, 0)$

$(0, 1)$

$[1, 2)$

$[2, \infty)$

Properties of Limits

Assume $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ exist and are finite.

(1) $\lim_{x \to c} (b \cdot f(x)) = b \cdot \lim_{x \to c} f(x)$ [for any constant b]

$\lim_{x \to c} (3 \cdot f(x))$

$= 3 \cdot \lim_{x \to c} f(x)$

(2) $\lim_{x \to c} (f(x) + g(x)) = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$

"the limit of the sum is the sum of the limits"

(3) $\lim_{x \to c} (f(x) g(x)) = \left(\lim_{x \to c} f(x)\right)\left(\lim_{x \to c} g(x)\right)$

(4) $\lim_{x \to c} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$, as long as the denom. isn't zero

(5) $\lim_{x \to c} p(x) = p(c)$ for any polynomial $p(x)$ (Why?)

Polynomials are continuous

finite = (not $+\infty$ or $-\infty$)

$$\lim_{x\to 2} \frac{(x+5)\cdot\cos(x)}{\sqrt{x}}$$

$$= \frac{\lim_{x\to 2}\left((x+5)\cdot\cos(x)\right)}{\lim_{x\to 2}\sqrt{x}} \quad \text{(Rule 4)}$$

continuous at $x=2$

$$= \frac{\left[\lim_{x\to 2}(x+5)\right]\cdot\left[\lim_{x\to 2}\cos(x)\right]}{\lim_{x\to 2}\sqrt{x}} \quad \text{(Rule 3)}$$

$$= \frac{(2+5)\cdot\cos(2)}{\sqrt{2}} = \frac{7\cos(2)}{\sqrt{2}}$$

Properties of Continuity

If $f(x)$ and $g(x)$ are continuous on an interval and b is a constant, then the following are continuous on the same interval:

* $b \cdot f(x)$
* $f(x) + g(x)$
* $f(x) \cdot g(x)$
* $\dfrac{f(x)}{g(x)}$, as long as $g(x) \neq 0$ anywhere in the interval

More continuity

- If $f(x)$ and $g(x)$ are continuous everywhere in their domain, then

  $f(g(x))$ and $g(f(x))$

  are also continuous in their domains.

- If $f(x)$ is continuous and if it has an inverse function $f^{-1}(x)$, then

  $f^{-1}(x)$ is continuous.

1.8

<u>Suggested HW</u>: 1, 2, 3, 6, 7, 8, 9, 10, 11, 15, 17, 19, 21, 23, 27, 28, 37, 44, 45, 47, 65, 67

Section 1.9 – Further Limit Calculations Using Algebra

We'll focus in this section on limits of functions of the form $\frac{f(x)}{g(x)}$ at $y = \pm\infty$ and at points where $g(x) = 0$.

potentially horizontal asymptotes

potentially vertical asymptotes

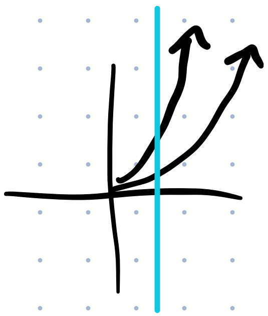
(1) $\lim_{x\to\infty} \frac{f(x)}{g(x)}$ and $\lim_{x\to-\infty} \frac{f(x)}{g(x)}$

$$\lim_{x\to\infty} \frac{f(x)}{g(x)}$$

If $g(x)$ "grows faster" than $f(x)$ as $x \to \infty$ or $-\infty$, then the limits are 0.

Examples

Which grows faster as $x \to \infty$?



x^6 vs. x^3

$-x^6$ vs x^3 (ignore + and − signs)

x^3+2x+1 vs $1+x^2+x^4$

$x^{1/2}=\sqrt{x}$ vs x

As $x \to \infty$

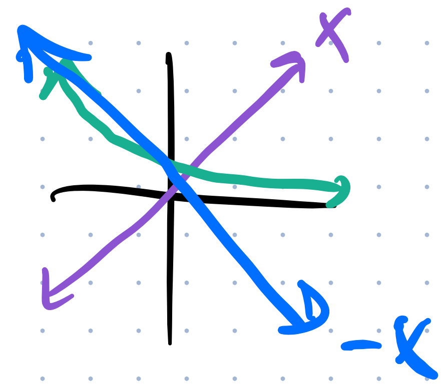
exp. growth

$(1.1)^x$ vs x^{1000}

polynomial

(eventually)

$\left(\frac{1}{2}\right)^x$ vs. x



As $x \to -\infty$

$\left(\frac{1}{2}\right)^x$ vs. x