Math 1450 – Calculus 1

Wed, Sept. 10

Announcements:

* HW 2 due tomorrow, 11:59pm
  covers 1.7 and part of 1.8
* Q2 tomorrow, in discussion in this room
  covers sugg. homework from last Fri, Monday, and today
* Exam 1 – Wednesday, Sept. 17, 5pm-6pm
  study guide on course website! 1.1-1.9

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! 12-1

Today:

→ 1.7: Introduction to Limits + Continuity

→ 1.8: Extending the Idea of a Limit

Limits

$$\text{"}\lim_{x \to c} f(x)\text{"}$$

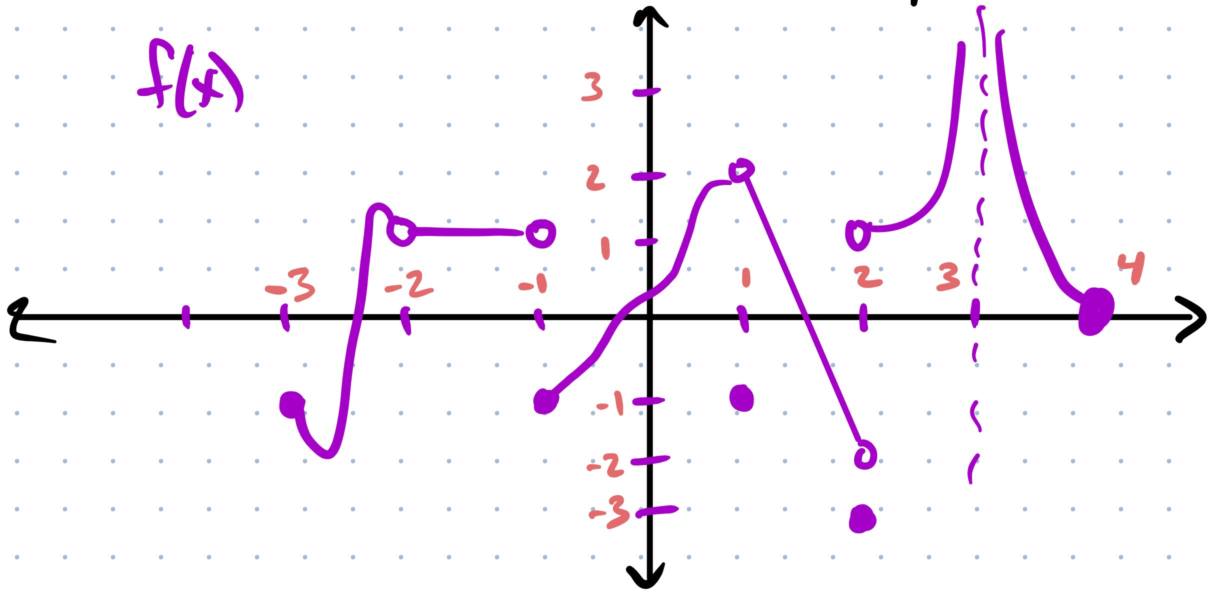
= "the limit of $f(x)$ as x approaches c"

= "what does it look like $f(c)$ should be if we:

(1) look at $f(x)$ at x-values close to $x=c$

(2) completely ignore $f(c)$ itself"

Group Work: Find each limit, if it exists.



$\lim_{x \to -3} f(x)$ DNE

$\lim_{x \to -2} f(x)$ 1

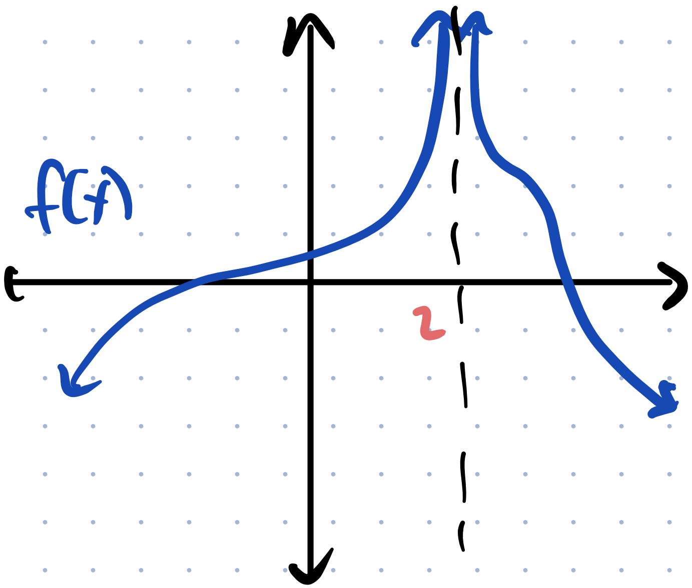
$\lim_{x \to -1} f(x)$ DNE

$\lim_{x \to 3} f(x)$ DNE / ∞

$\lim_{x \to 2} f(x)$ DNE

$\lim_{x \to 1} f(x)$ 2

WARNING: The book and Wiley Plus are not always consistent with limits that are $\pm\infty$.



Sometimes they will say

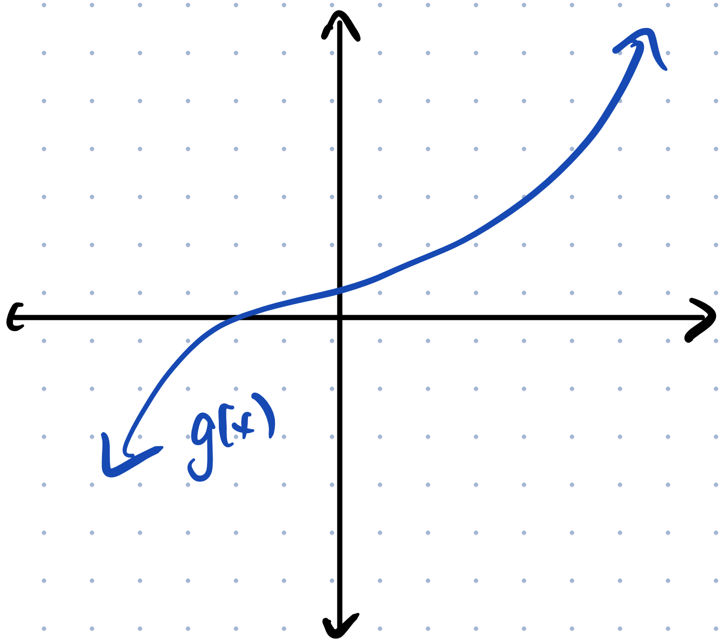
$$\lim_{x\to 2} f(x) = \text{DNE}$$

other times they will say

$$\lim_{x\to 2} f(x) = \infty$$

better

Same with:

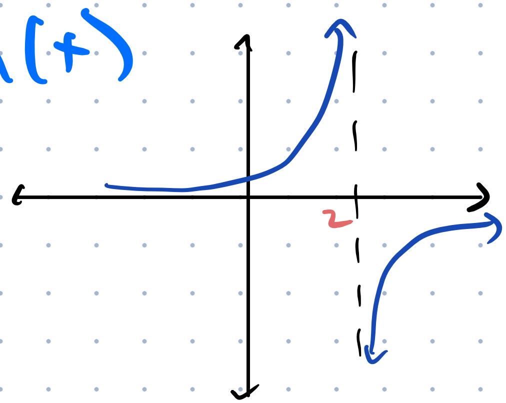


$$\lim_{x \to \infty} g(x) = \infty \text{ or DNE}$$

$$\lim_{x \to -\infty} g(x) = -\infty \text{ or DNE}$$

For this one, DNE is the only answer:

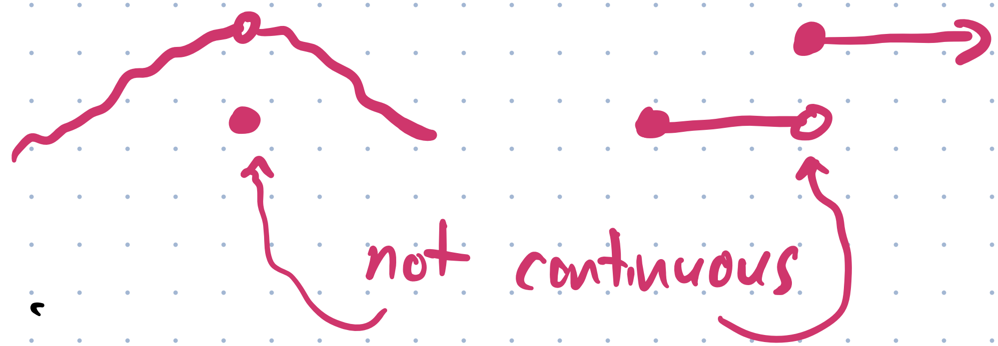
h(x)



$$\lim_{x \to 2} h(x) = \underline{\text{DNE}}$$

Continuity Again

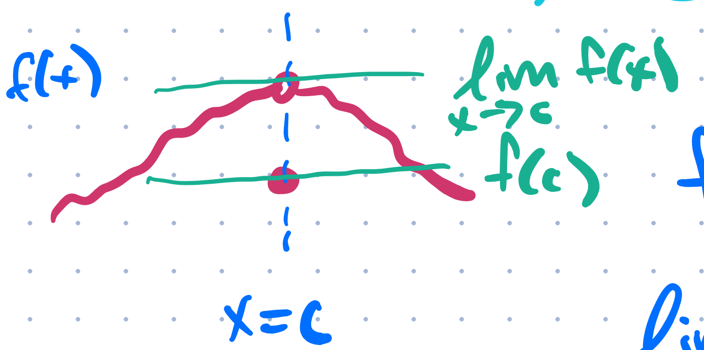
More precise definition!



The function $f(x)$ is continuous at the point $x=c$ if:

(1) $f(c)$ exists

(2) $\lim_{x \to c} f(x) = f(c)$



$f(c)$

$\lim_{x \to c} f(x)$

"The value at c is exactly what the nearby points suggest it should be. They don't trick you."

Calculating Limits: many techniques, depending on the function

$$\lim_{x \to c} f(x) = L$$

(1) If you have the graph, just look at it.

(2) If you know that f(x) is continuous at $x=c$, just plug in c: $L = f(c)$

(3) If you have a calculator, plug in x-values really close to c on the left and right.

$\lim_{x \to 1} f(x)$

$x = 0.9$
0.99
0.99999

$x = 1.1$
$x = 1.01$
$x = 1.00001$

(~~3~~ 4) Sometimes we can do some algebra.

$$\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{(x-3)(x+3)}{x-3}$$

$a^2 - b^2$

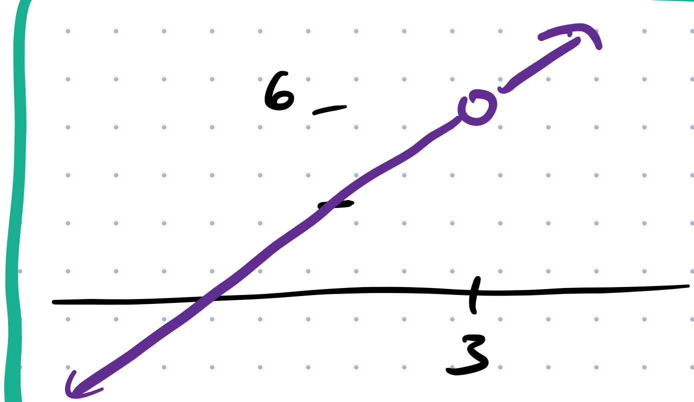
$(a+b)(a-b)$

$$\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = x+3 \quad \left(\begin{matrix}\text{as long} \\ \text{as } x \neq 3\end{matrix}\right)$$

But remember, when finding the limit at $x=3$, we don't care what actually happens at $x=3$.

So, $\displaystyle\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} (x+3) = 3+3$

$= 6$

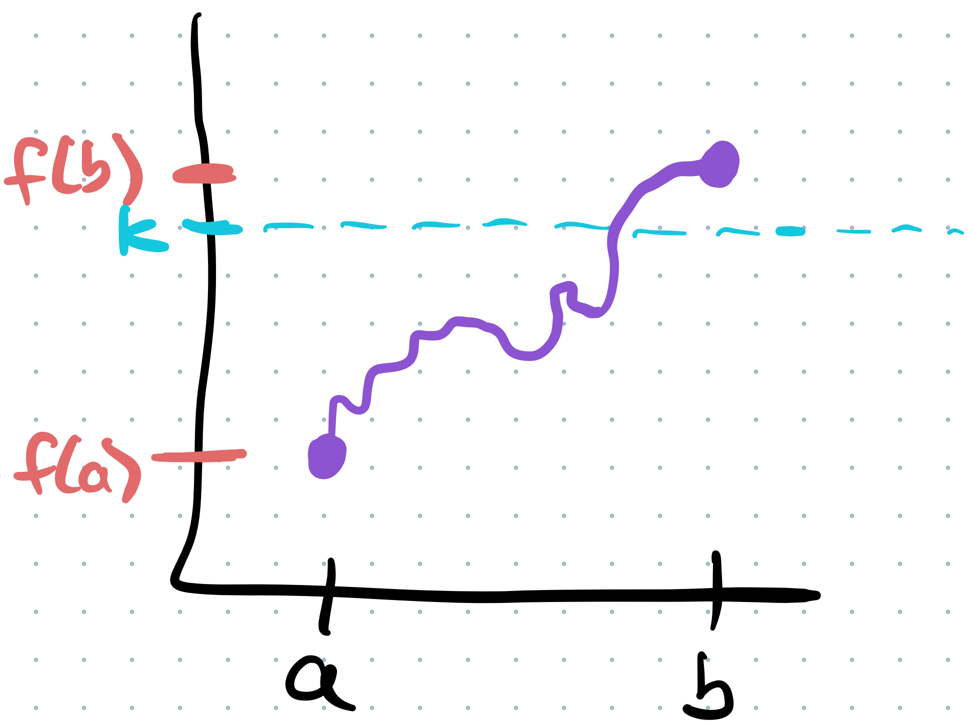


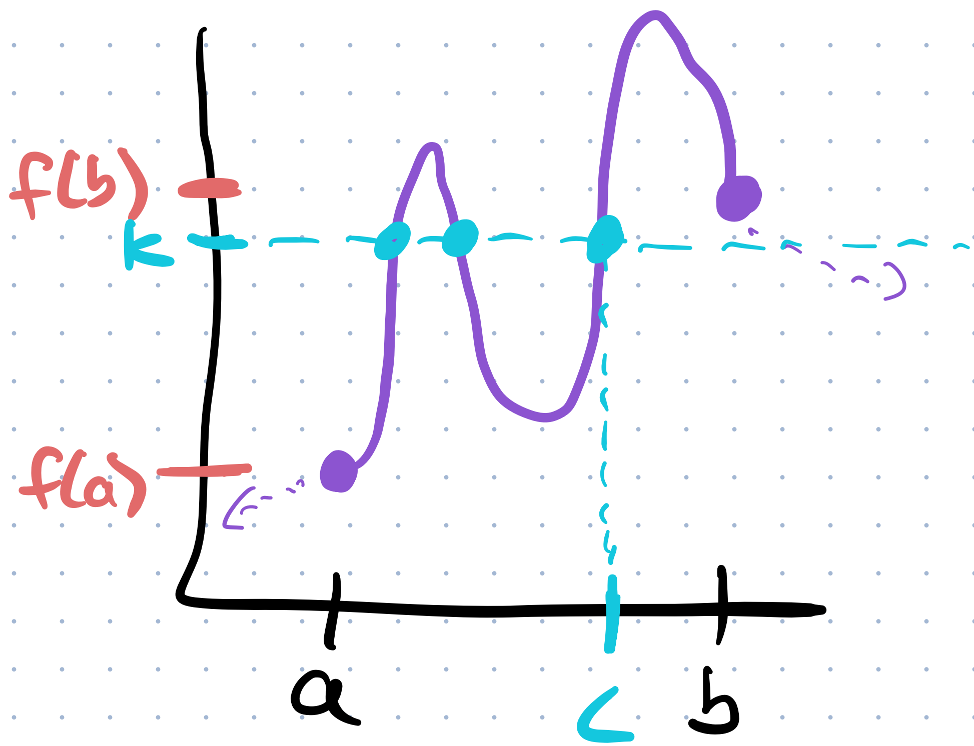
$$\lim_{x \to 5} \frac{x^2 - 2x - 15}{x - 5} = \lim_{x \to 5} \frac{\cancel{(x-5)}(x+3)}{\cancel{x-5}}$$

$$= \lim_{x \to 5} (x+3) = 5 + 3 = 8$$

Intermediate Value Theorem

Suppose $f(x)$ is continuous on the interval $[a,b]$. Then, for any number k between $f(a)$ and $f(b)$, there exists some point c in $[a,b]$ such that $f(c)=k$.





Ex:

If your drink was 70° when you put it in the fridge and 30° when you took it out, then at some time in the middle, it was exactly 40°.

Suggested Homework: 1-9, 11, 13, 15, 17, 19-21, 23, 24, 25, 27, 28, 31, 33, 35, 43, 45, 57, 59, 89

1.7

Section 1.8 - Extending the Idea of a Limit

In 1.7: $\lim_{x \to 2} f(x) =$ the # that it looks like $f(2)$ should be if you look at the x-values around 2, but ignore $x=2$.

In 1.8: one-sided limits

ignore points to the left of 2

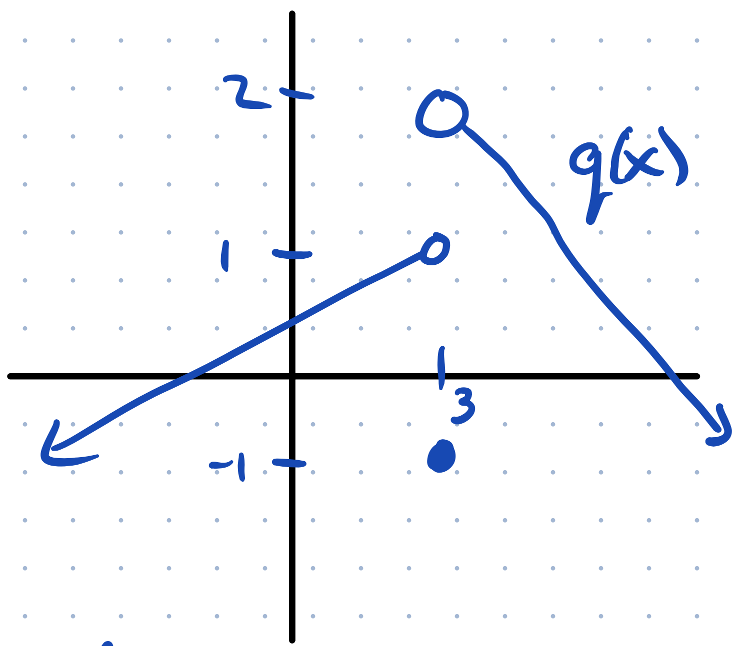
$\lim_{x \to 2^+} f(x) =$ what it looks like $f(2)$ should be ignoring what it actually is and only looking at nearby points to the right of 2. (> 2)

"from the right"

$\lim_{x \to 2^-} f(x) =$ same, but look at points to the left

ignore right

Ex:



$g(3) = -1$

$\lim_{x \to 3^+} g(x) = 2$

$\lim_{x \to 3^-} g(x) = 1$

$\lim_{x \to 3} g(x) = \text{DNE}$

Ex: Find all three kinds of limits at $x=2$ for $\frac{|x-2|}{x-2}$

$$|a| = \begin{cases} a, & \text{if } a \geq 0 \\ -a, & \text{if } a < 0 \end{cases}$$

$|3| = 3 \quad (a=3)$

$|-3| = -(-3) = 3$

$$|x-2| = \begin{cases} x-2 & \text{if } (x-2) \geq 0 \\ -(x-2) & \text{if } (x-2) < 0 \end{cases} = \begin{cases} x-2 & \text{if } x \geq 2 \\ -(x-2) & \text{if } x < 2 \end{cases}$$

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} & \text{if } x > 2 \\ \frac{-(x-2)}{x-2} & \text{if } x < 2 \end{cases} = \begin{cases} 1, & x > 2 \\ -1, & x < 2 \end{cases}$$

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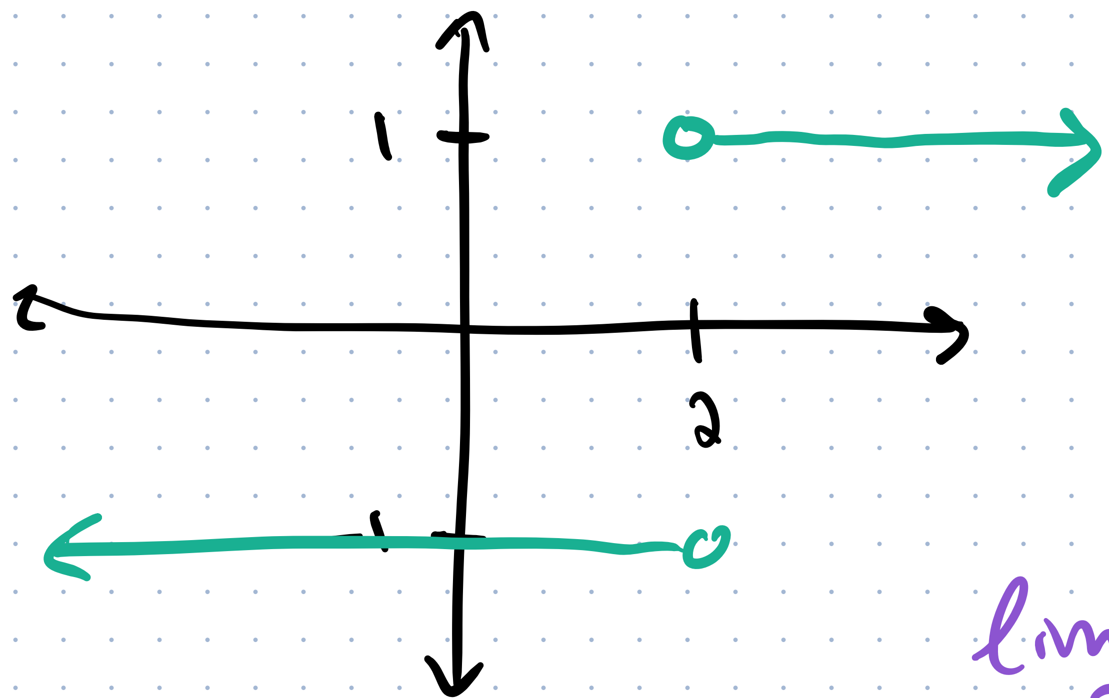

$$\lim_{x\to 2^+} \frac{|x-2|}{x-2} = 1$$

$$\lim_{x\to 2^-} \frac{|x-2|}{x-2} = -1$$

$$\lim_{x\to 2} \frac{|x-2|}{x-2} = \text{DNE}$$