Math 1450 - Calculus 1

Mon, Sept. 8

Announcements:

* HW 2 due Thurs, Sept 11, 11:59pm
  covers 1.7 and part of 1.8
* Q 2 on Thurs, Sept. 11 in discussion
  covers sugg. homework from last Fri, today, and Wed.
* Exam 1 - Wednesday, Sept. 17, 5pm-6pm

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk! (hours changed!)

Today:

→ 1.7: Introduction to Limits + Continuity

→ 1.8: Extending the Idea of a Limit

Section 1.7 - Introduction to Limits and Continuity

All of Calculus is built on top of the concept of a limit.

"How does a function behave as you get closer and closer to some point?"

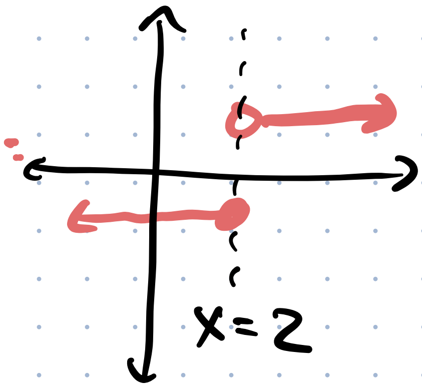
Continuity

A function is <u>continuous at a particular point</u> if:

(1) it exists at that point

(2) there are no jumps or breaks at that point.

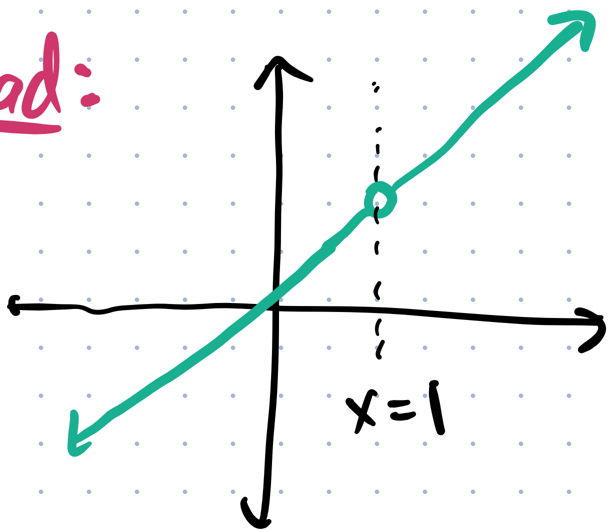
Bad:



Not continuous at $x=2$.

It exists at $x=2$, but it has a jump

Bad:



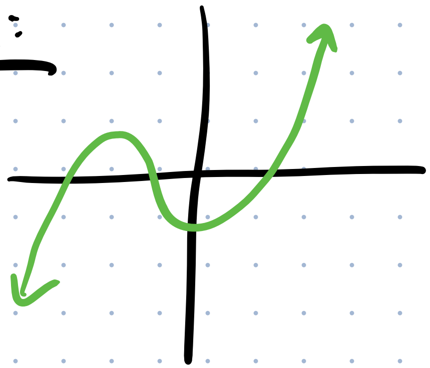
$f(1)$ does not exist,
so it's not continuous at $x=1$

Continuity

A function is continuous in a whole interval if:

it is continuous at every point in the interval

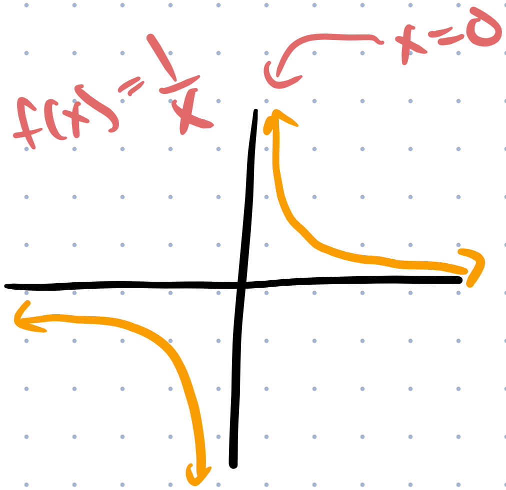
Exs:



continuous at every point

continuous on the interval

$(-\infty, \infty)$

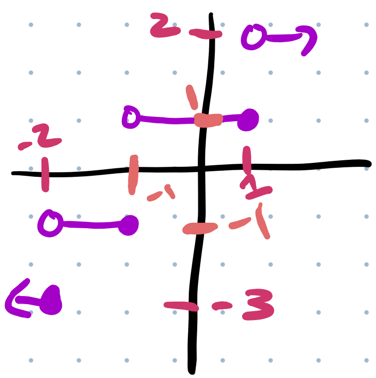


not continuous at $x=0$ (D.N.E.)

is continuous at all other points

continuous on the intervals

$(-\infty, 0)$ and $(0, \infty)$



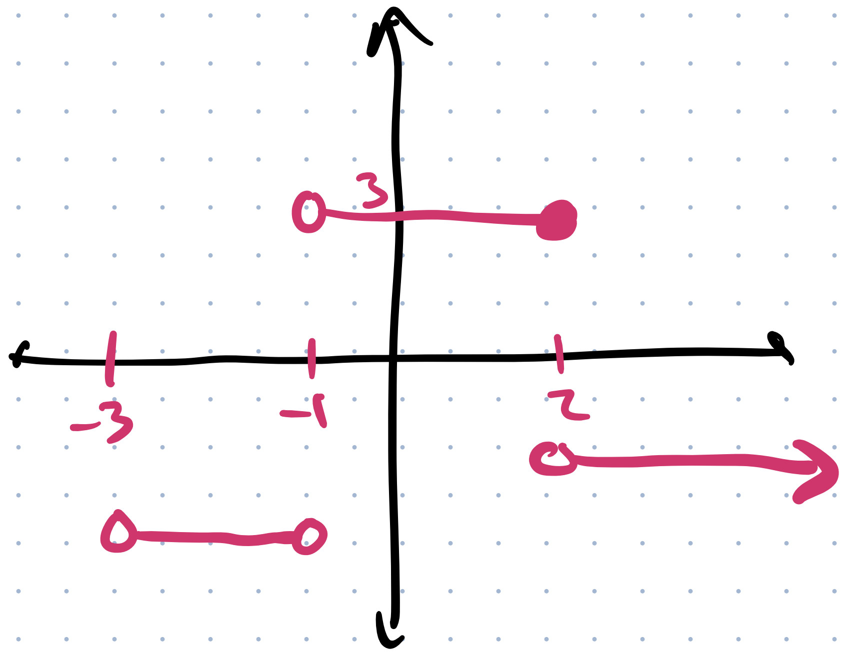
exists everywhere

jumps at $x = -2, -1, 1$

not continuous at those x-values

continuous on the intervals

$(-\infty, -2)$ and $(-2, -1)$ and $(-1, 1)$ and $(1, \infty)$



$f(-1)$ DNE

$f(2) = 3$

continuous in the intervals $(-3,-1)$

$(-1,2)$ $(2,\infty)$

Continuity

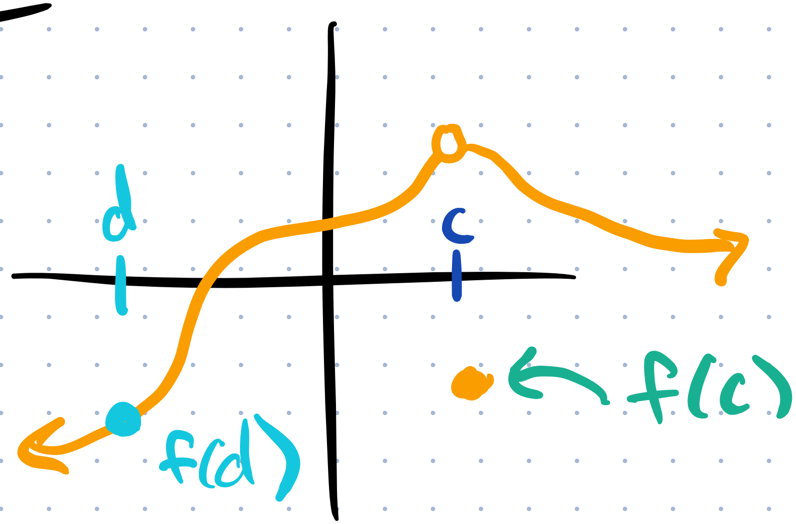
Another way to say "$f(x)$ is continuous at $x=c$" is:

"The y-values of $f(x)$, when x is very close to c, are very close to $f(c)$."

OR

"As x approaches c, $f(x)$ approaches $f(c)$."

Ex:



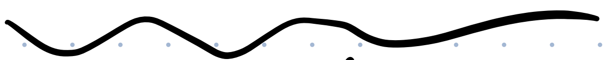
not continuous at $x=c$

As $x \to c$, does $f(x) \to f(c)$?

No.

Which functions are continuous?

- Exponentials
- Polynomials
- Sine + Cos

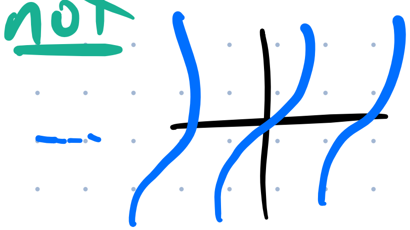


at all points

- Logarithms
- Tangent
- Rational Functions

Only on their domain

These don't exist at some points — definitely <u>not</u> continuous there.



But any points where they <u>do</u> exist, they are continuous.

Limits

"$\lim_{x \to c} f(x)$"

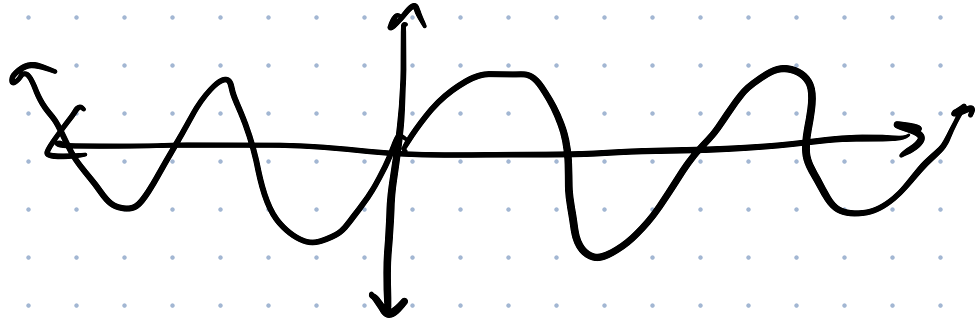
= "the limit of f(x) as x approaches c"

= "what does it look like f(x) <u>should be</u> if we:

(1) look at f(x) at x-values <u>close</u> to x=c

(2) completely ignore f(c) itself"

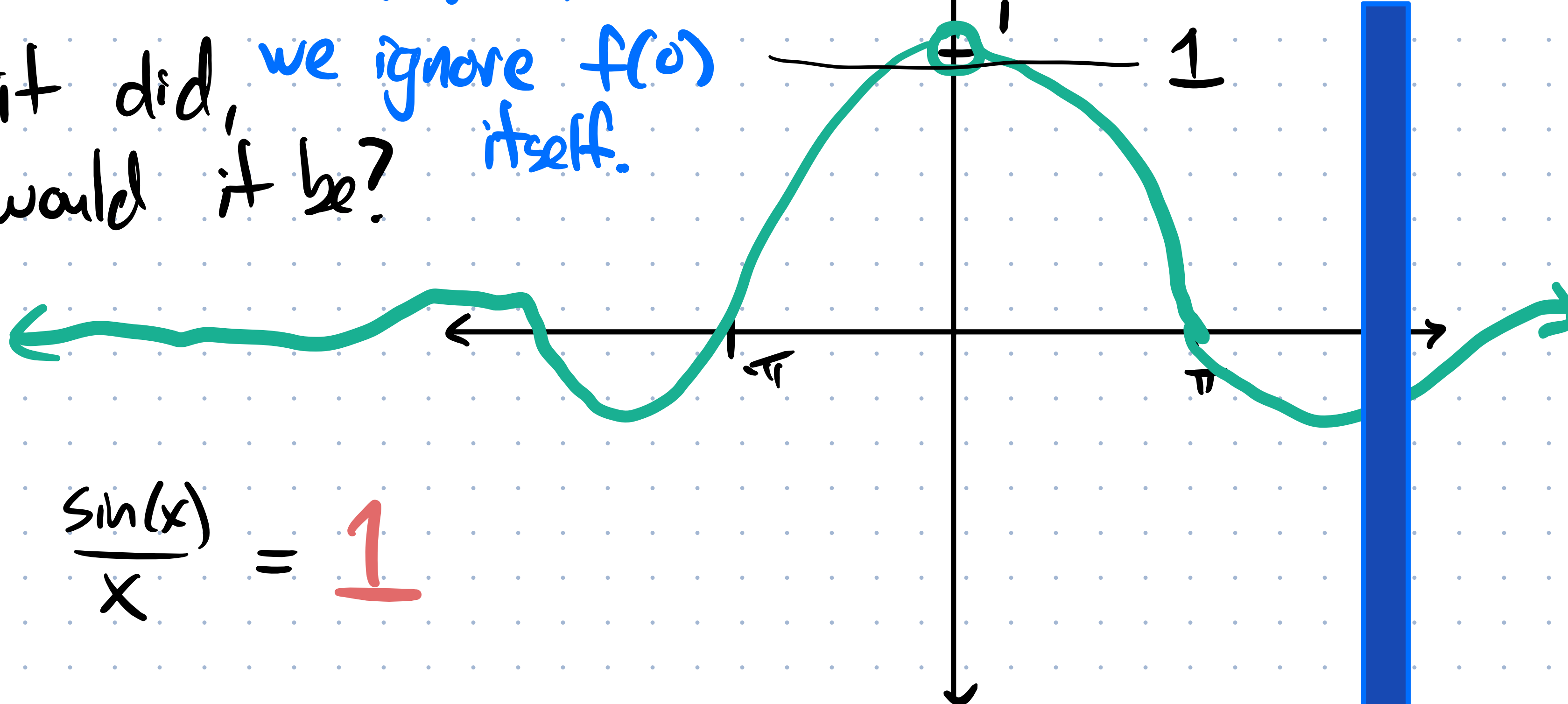
Ex: Let $f(x) = \dfrac{\sin(x)}{x}$



$f(0)$ does not exist!

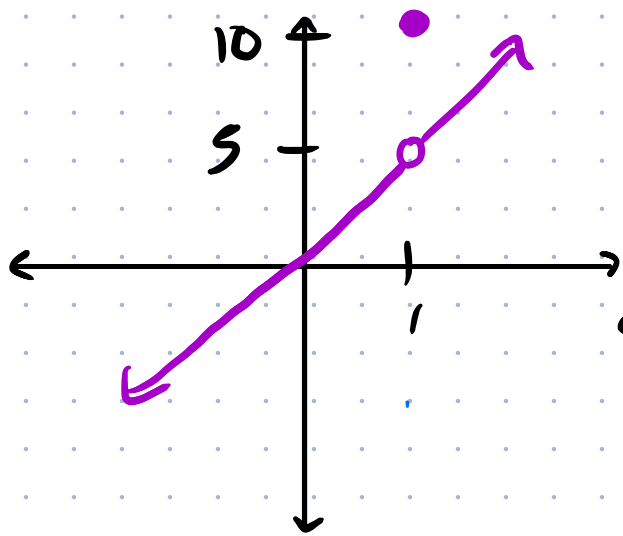
But $f(x)$ looks like we should have $f(0) = 1$ if we ignore $f(0)$ itself.

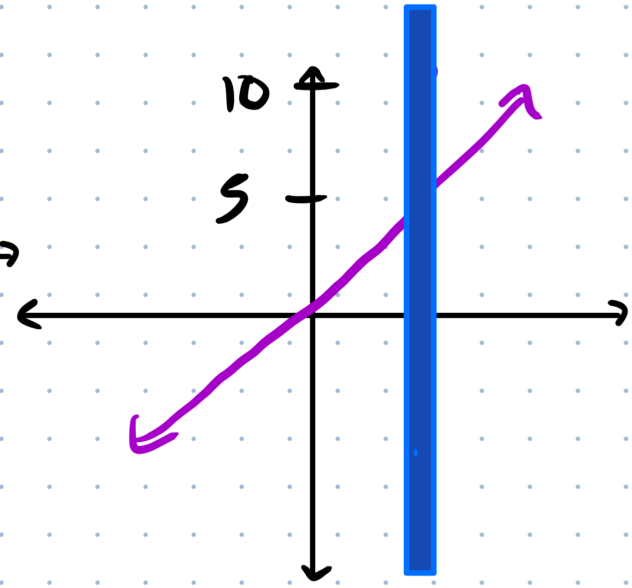
But if it did, what would it be?



So, $\displaystyle\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

Ex: Let f(x) =

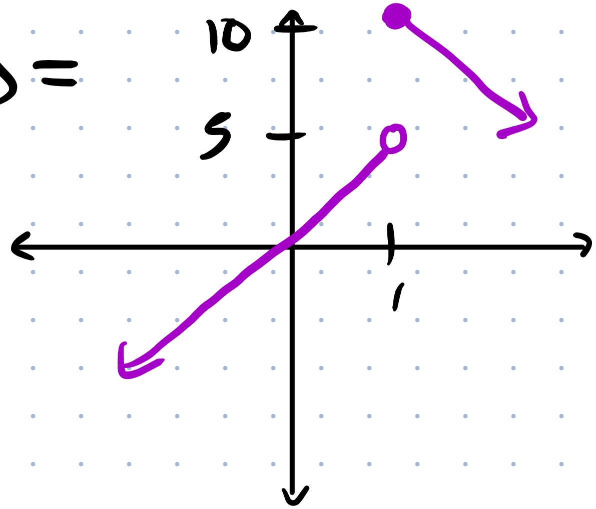


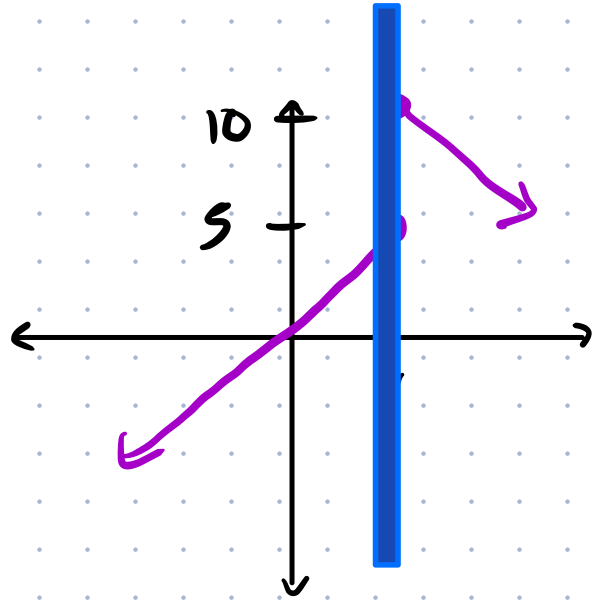


f(1) exists, and f(1) = 10

But the limit does not care what actually happens at x = 1.

So, $\lim_{x \to 1} f(x) = 5$.

Ex: Let $f(x) =$ 

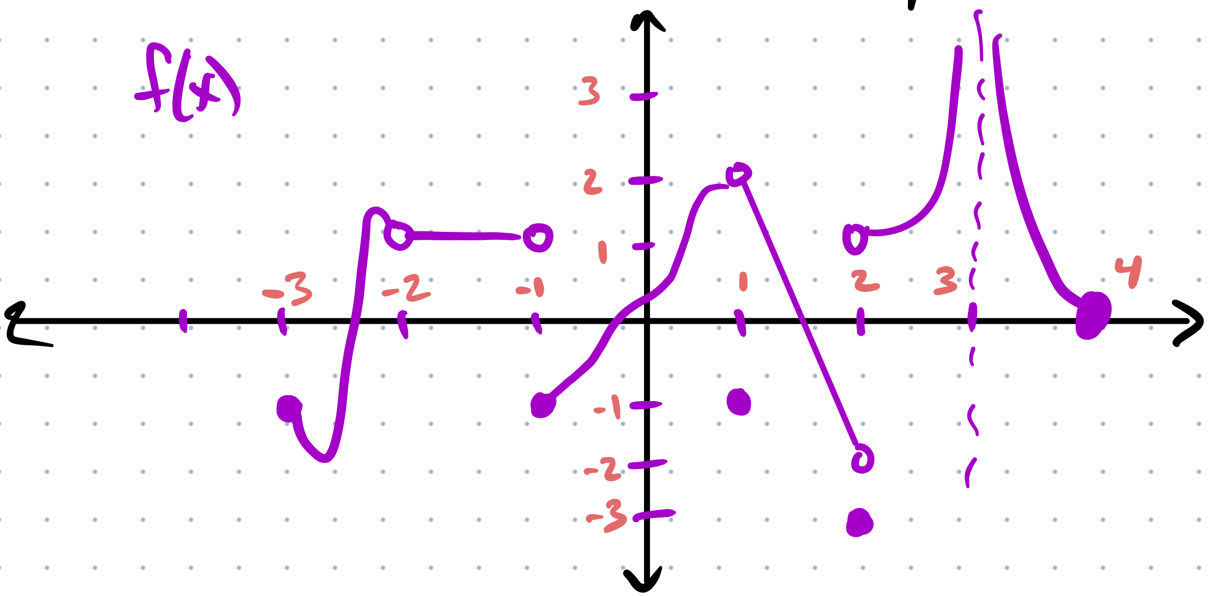


What is $\lim_{x \to 1} f(x)$?

One side makes it look like the limit should be 5.
The other side makes it look like 10.

Ambiguous $\rightsquigarrow$ The limit does not exist.
(DNE)

Group Work: Find each limit, if it exists.



$\lim_{x \to -3} f(x)$ DNE

$\lim_{x \to -2} f(x)$ 1

$\lim_{x \to -1} f(x)$ DNE

$\lim_{x \to 3} f(x)$ DNE / ∞

$\lim_{x \to 2} f(x)$ DNE

$\lim_{x \to 1} f(x)$ 2