Math 1450 - Calculus 1

Fri, Sept. 5

Announcements:

- * HW 1 due Sunday, Sept. 7, 11:59pm
- * HW 2 due Thurs, Sept 11, 11:59pm
  covers 1.7 and part of 1.8
- * Q2 on Thurs, Sept. 11 in discussion
  covers sugg. homework from today
  and next Mon + Wed

Office Hours

Mondays, 12-1

Wednesdays, 2-3

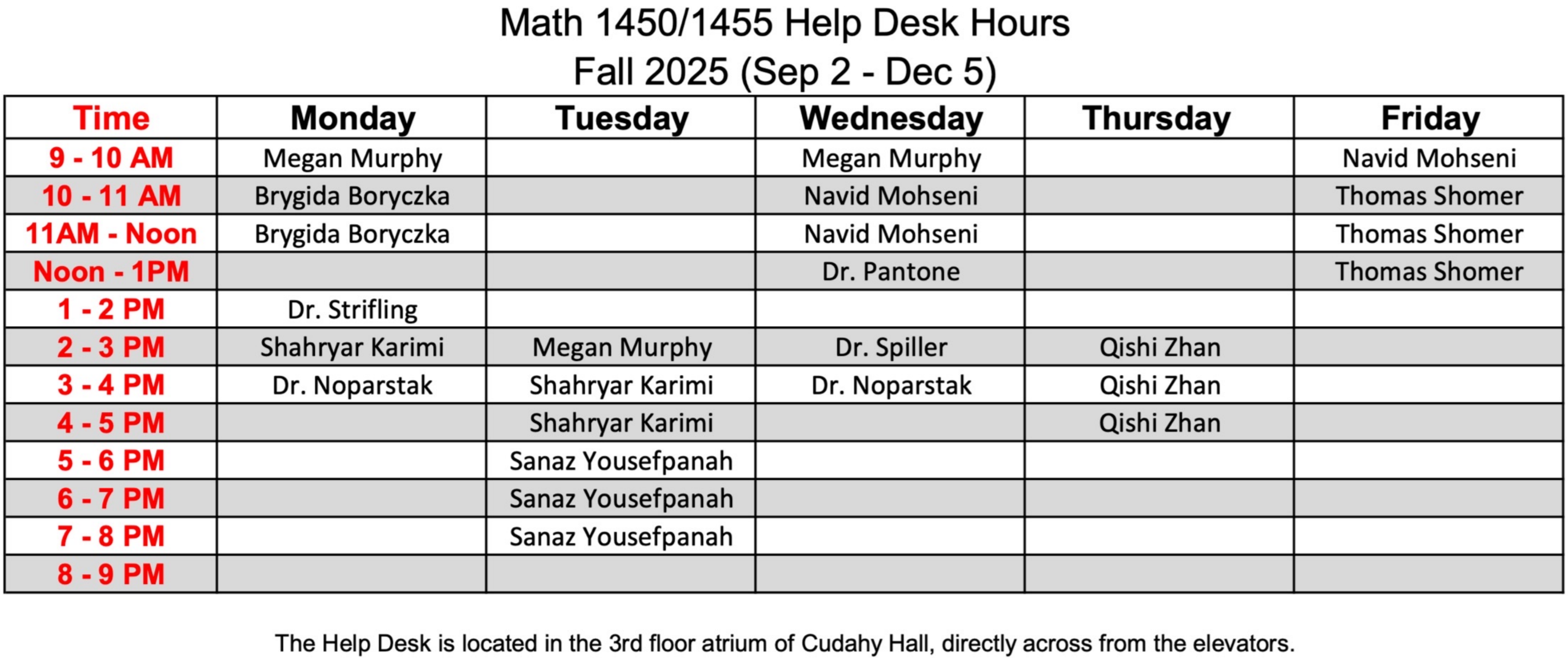
+ Help Desk!

Today:

→ 1.5: Trigonometric Functions

→ 1.6: Powers and Polynomials

The Help Desk is now open!



Math 1450/1455 Help Desk Hours
Fall 2025 (Sep 2 - Dec 5)

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9 - 10 AM	Megan Murphy		Megan Murphy		Navid Mohseni
10 - 11 AM	Brygida Boryczka		Navid Mohseni		Thomas Shomer
11AM - Noon	Brygida Boryczka		Navid Mohseni		Thomas Shomer
Noon - 1PM			Dr. Pantone		Thomas Shomer
1 - 2 PM	Dr. Strifling				
2 - 3 PM	Shahryar Karimi	Megan Murphy	Dr. Spiller	Qishi Zhan	
3 - 4 PM	Dr. Noparstak	Shahryar Karimi	Dr. Noparstak	Qishi Zhan	
4 - 5 PM		Shahryar Karimi		Qishi Zhan	
5 - 6 PM		Sanaz Yousefpanah			
6 - 7 PM		Sanaz Yousefpanah			
7 - 8 PM		Sanaz Yousefpanah			
8 - 9 PM					

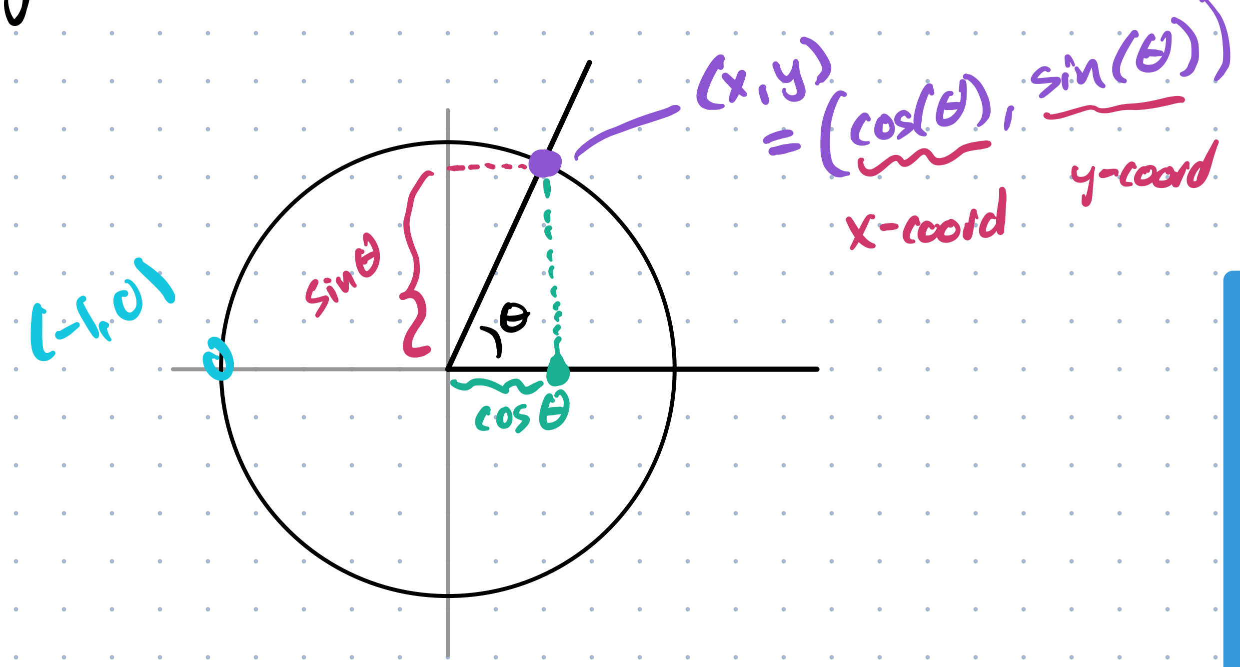
The Help Desk is located in the 3rd floor atrium of Cudahy Hall, directly across from the elevators.

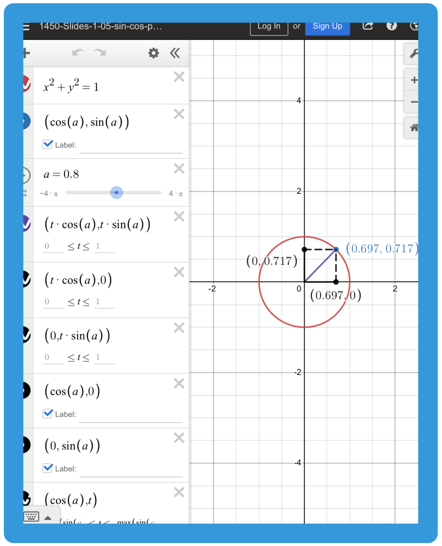
You can come to any of these scheduled times.

3rd floor of Cudahy, table near the bathrooms.

Sine and cosine

As you spin a point around the outside of a circle, the trig functions cos and sin tell you how the x and y coordinates of that point change.





$x^2 + y^2 = 1$

$(\cos(a), \sin(a))$

☑ Label:

$a = 0.8$

$-4 \cdot \pi$ … $4 \cdot \pi$

$(t \cdot \cos(a), t \cdot \sin(a))$

$0 \le t \le 1$

$(t \cdot \cos(a), 0)$

$0 \le t \le 1$

$(0, t \cdot \sin(a))$

$0 \le t \le 1$

$(\cos(a), 0)$

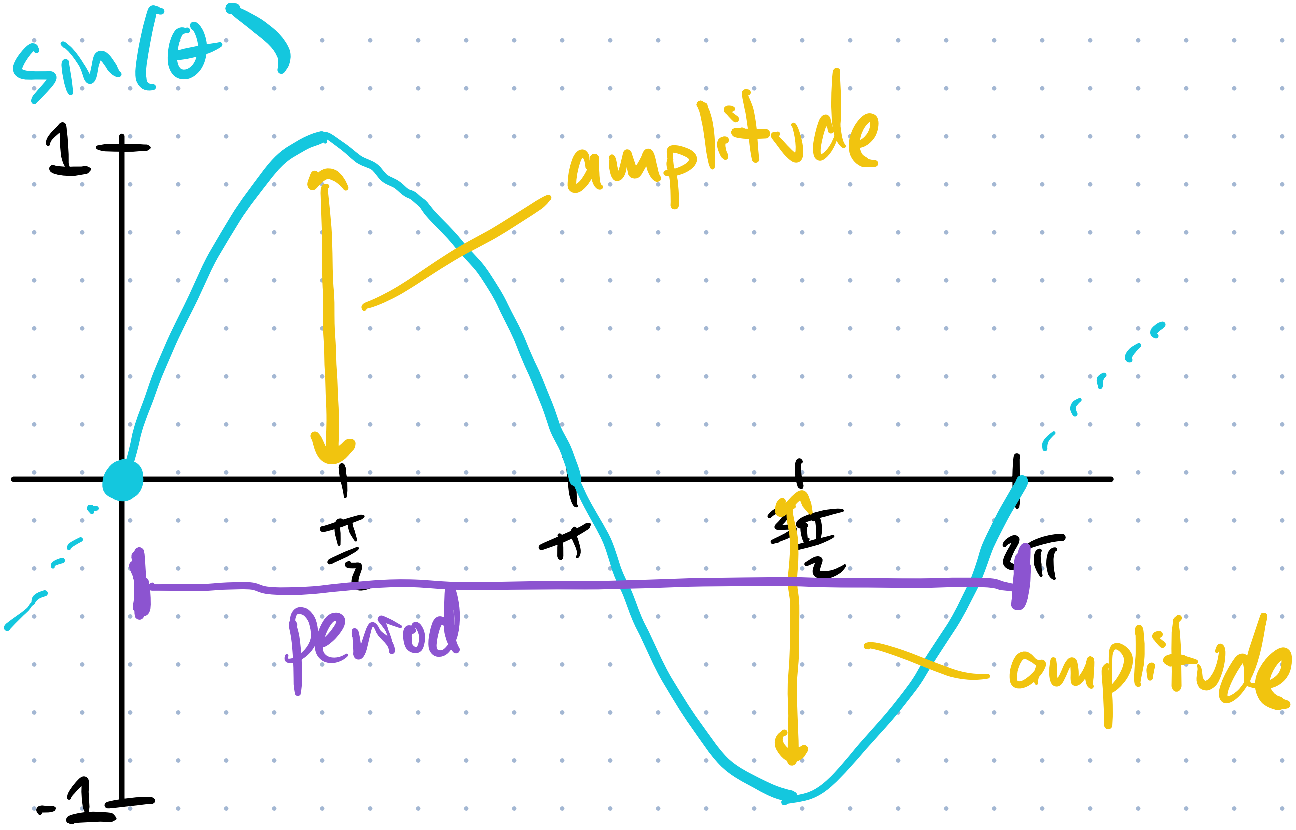
☑ Label:

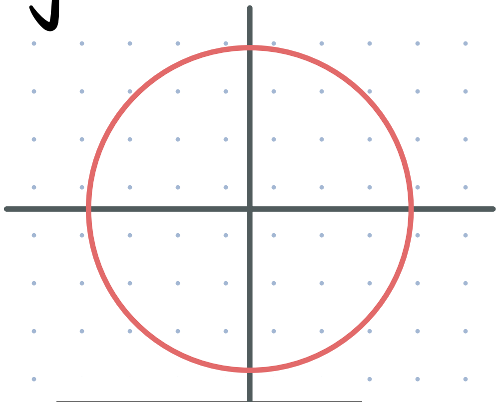
$(0, \sin(a))$

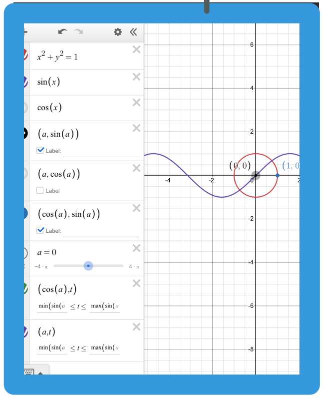
☑ Label:

$(\cos(a), t)$

sin(θ) : y-coordinate of a point going around a circle







$x^2 + y^2 = 1$

$\sin(x)$

$\cos(x)$

$(a, \sin(a))$

☑ Label:

$(a, \cos(a))$

☐ Label

$(\cos(a), \sin(a))$

☑ Label:

$a = 0$

$-4 \cdot \pi$ … $4 \cdot \pi$

$(\cos(a), t)$

$\min(\sin(a \le t \le \max(\sin(a$

(a, t)

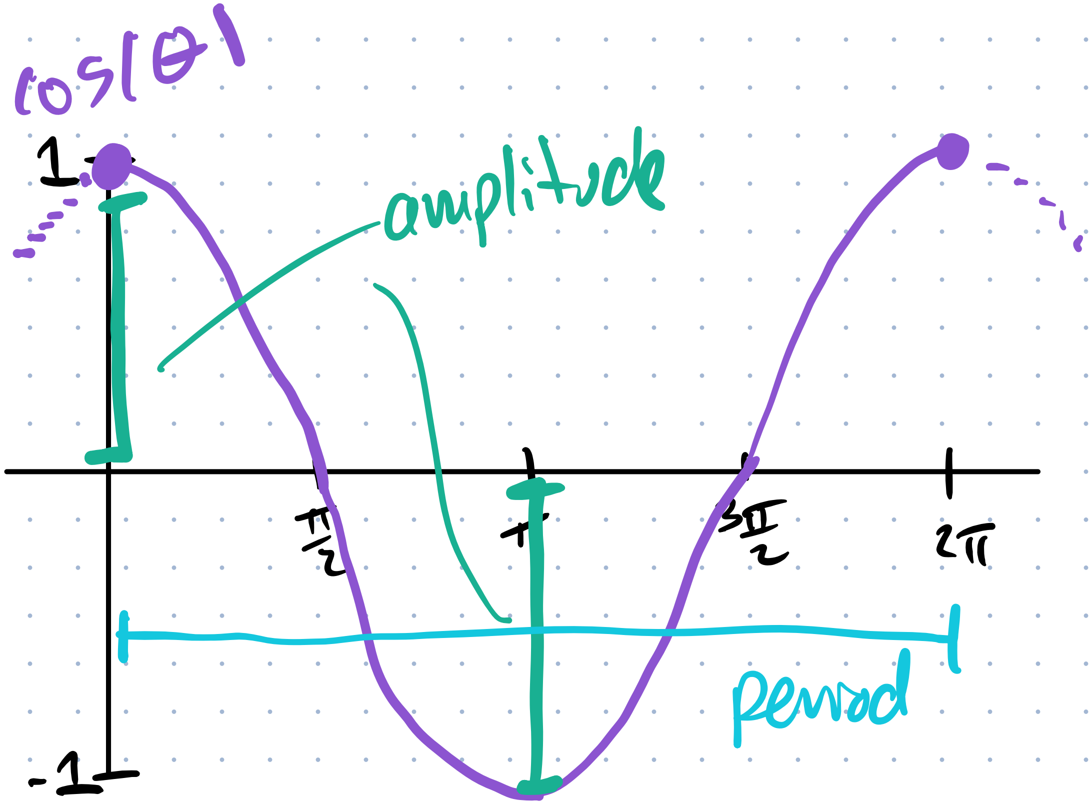
$\min(\sin(a \le t \le \max(\sin(a$

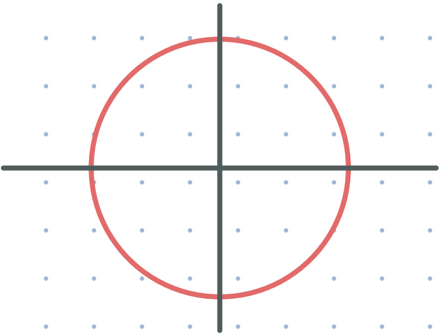
amplitude = 1

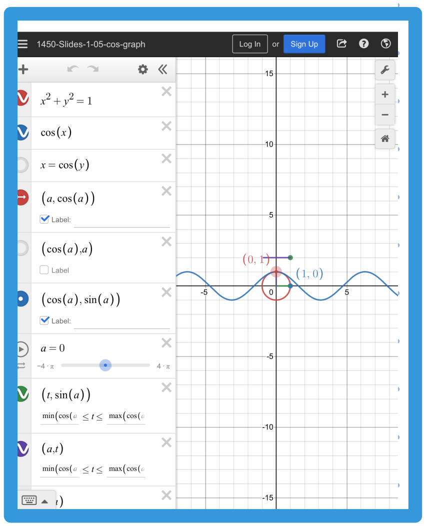
period = 2π

(how long until it repeats

$\cos(\theta)$: x-coordinate of a point going around a circle







$x^2 + y^2 = 1$

$\cos(x)$

$x = \cos(y)$

$(a, \cos(a))$

☑ Label:

$(\cos(a), a)$

☐ Label

$(\cos(a), \sin(a))$

☑ Label:

$a = 0$

$-4 \cdot \pi$ $4 \cdot \pi$

$(t, \sin(a))$

$\min(\cos($ $\le t \le$ $\max(\cos($

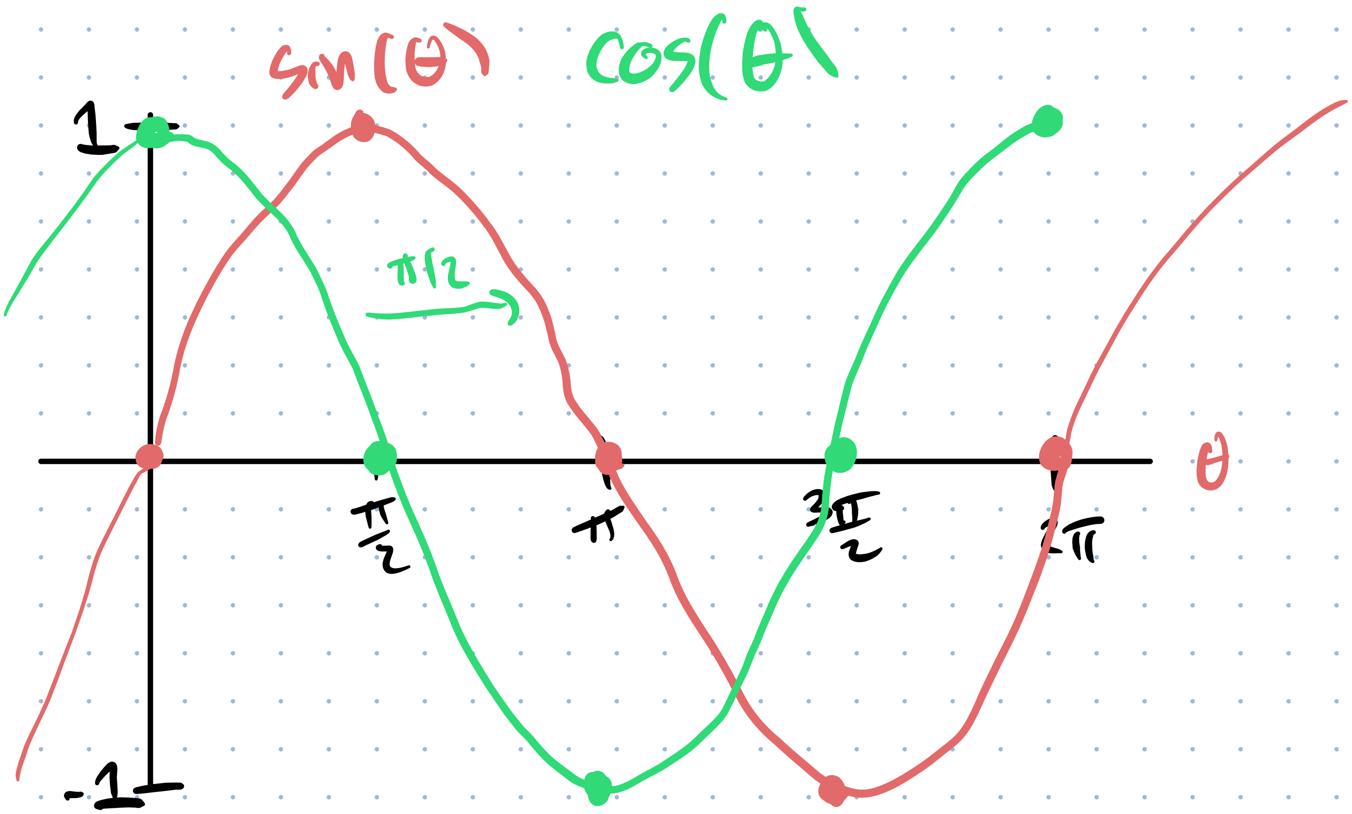
(a, t)

$\min(\cos($ $\le t \le$ $\max(\cos($

amplitude = 1

period = 2π

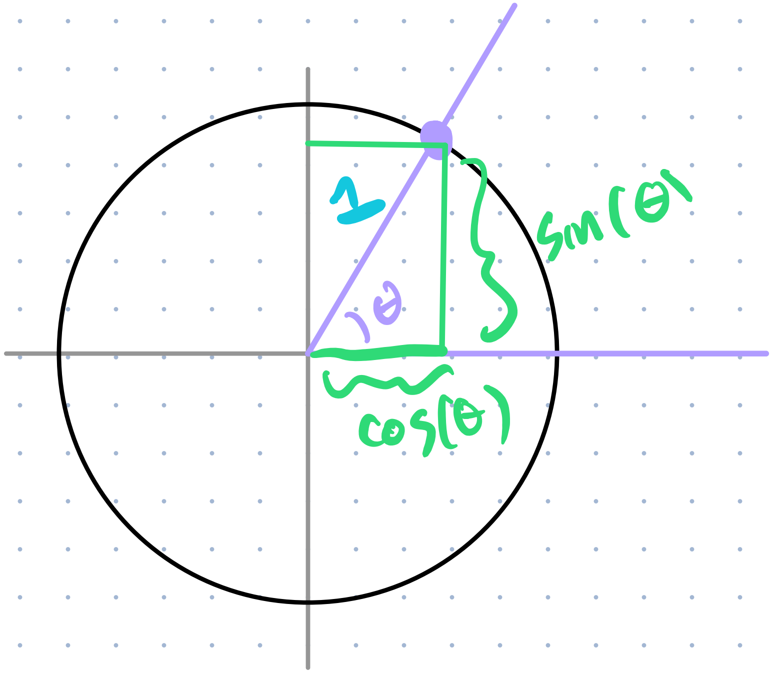
sine and cosine together



$$\underline{\sin}(\theta) = \underline{\cos}\left(\theta - \frac{\pi}{2}\right)$$

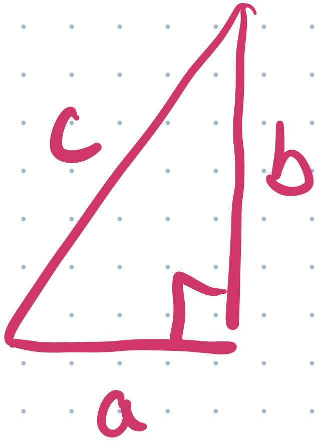
Notation: Instead of $(\sin(\theta))^2$, it is common to write $\sin^2(\theta)$.

Fact: For any angle θ: $\sin^2(\theta) + \cos^2(\theta) = 1$.



$$1^2 = \sin^2(\theta) + \cos^2(\theta)$$

pythagorean's theorem



$$c = \sqrt{a^2 + b^2}$$

$$c^2 = a^2 + b^2$$

sine and cosine can be transformed like any other function.

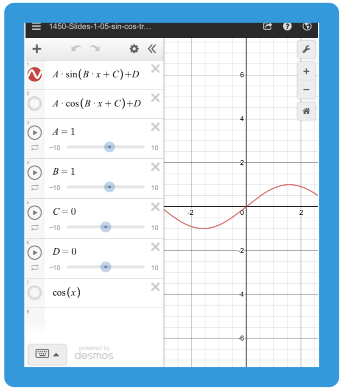
$$A \cdot \sin(B \cdot \theta)$$

amplitude $|A|$ period $\frac{2\pi}{|B|}$

$|\cdot|$ means "absolute value" make it positive

$|-3| = 3$

$|3| = 3$



1450-Slides-1-05-sin-cos-tr...

1. $A \cdot \sin(B \cdot x + C) + D$
2. $A \cdot \cos(B \cdot x + C) + D$
3. $A = 1$ −10 10
4. $B = 1$ −10 10
5. $C = 0$ −10 10
6. $D = 0$ −10 10
7. $\cos(x)$
8.

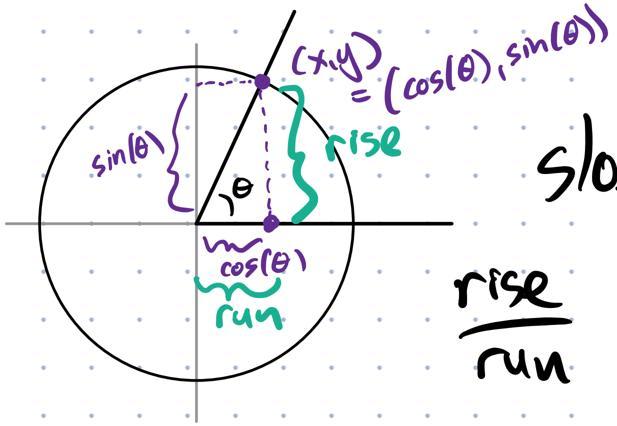
Lots of great pictures and examples in the book!

Tangent

The function $\tan(\theta)$ is defined as

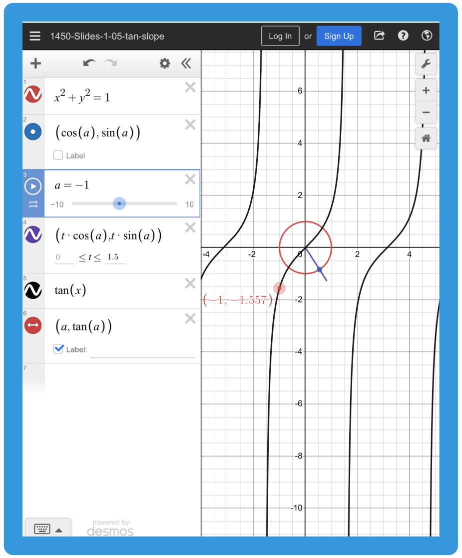
$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}.$$

$\tan(\theta)$ has a very nice visual interpretation.



slope of the line:

$$\frac{\text{rise}}{\text{run}} = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$$



1450-Slides-1-05-tan-slope

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1. $x^2 + y^2 = 1$
2. $(\cos(a), \sin(a))$ ☐ Label
3. $a = -1$ (−10 to 10)
4. $(t \cdot \cos(a), t \cdot \sin(a))$ $0 \le t \le 1.5$
5. $\tan(x)$
6. $(a, \tan(a))$ ☑ Label:

$\tan(0) = 0$ $\tan(\frac{\pi}{4}) = 1$ $\tan(\frac{\pi}{2}) =$ undefined

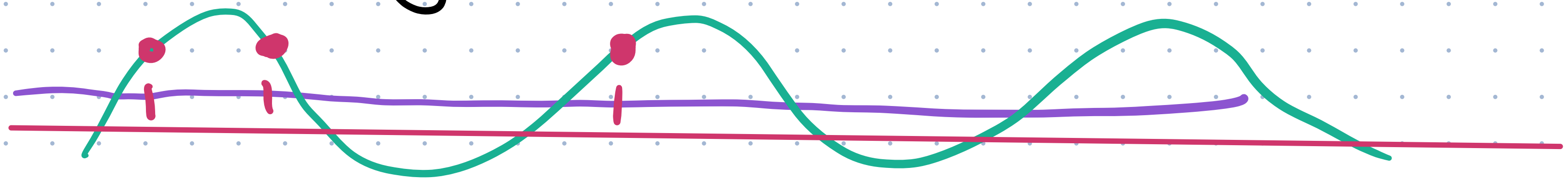
Period =

Amplitude =

Inverse trig functions

$3 = 2\sin(x+1)$

$\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$ don't automatically have inverses, because they all fail the horizontal line test.



To fix this, we restrict their domains to a smaller interval in which they do pass the horizontal line test.

~~$\sin^2$~~

$\sin : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ "$\sin^{-1}$" or "arcsin"

$\cos : [0, \pi]$ "$\cos^{-1}$" or "arccos"

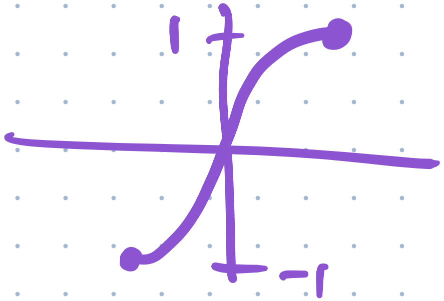
$\tan : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ "$\tan^{-1}$" or "arctan"

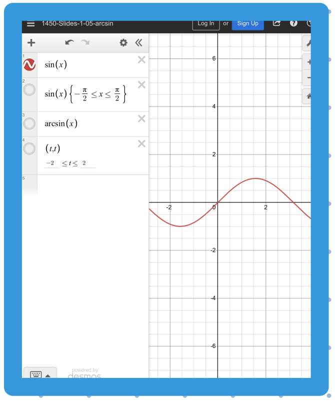
Ex: sin: domain $= [-\frac{\pi}{2}, \frac{\pi}{2}]$

range $= [-1, 1]$

arcsin: domain $= [-1, 1]$

range $= [-\frac{\pi}{2}, \frac{\pi}{2}]$





1 $\sin(x)$

2 $\sin(x)\left\{-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right\}$

3 $\arcsin(x)$

4 (t,t)

$-2 \le t \le 2$

arcsin(5)?

does not exist

Useful for solving equations involving trig functions.

<u>Ex</u>: Find <u>a solution</u> to $2\cos(x^2+1)-1=0$.

$$\Rightarrow 2\cos(x^2+1)=1$$

$$\Rightarrow \cos(x^2+1)=\frac{1}{2}$$

$$\Rightarrow \cancel{\arccos}(\cancel{\cos}(x^2+1))=\arccos(\tfrac{1}{2})$$

$$\Rightarrow x^2+1=\arccos(\tfrac{1}{2})$$

$$\Rightarrow x^2=\arccos(\tfrac{1}{2})-1$$

$$\Rightarrow x=\pm\sqrt{\arccos(\tfrac{1}{2})-1}\approx\pm 0.217$$

Reciprocal functions:

$\csc(x) = \frac{1}{\sin(x)}$ $\qquad$ $\sec(x) = \frac{1}{\cos(x)}$

"cosecant" $\qquad$ "secant"

$\cot(x) = \frac{1}{\tan(x)}$.

"cotangent"

$\arcsin(x)$

$=$

$\frac{1}{\sin(x)}$ is not the same as $\sin^{-1}(x)$!

$\neq$

$(\sin(x))^{-1}$

Suggested HW: 1-8, 10, 11, 15, 16, 20, 25, 26, 27, 29, 41, 43,
45, 63

Section 1.6 – Powers, Polynomials, and Rational Functions

A **power function** is a function of the form

$$f(x) = k \cdot x^p$$

variable

constants

Compare to exponential functions: $f(x) = k \cdot a^x$

Examples:

$3x^2$

$\pi x^{1/2}$

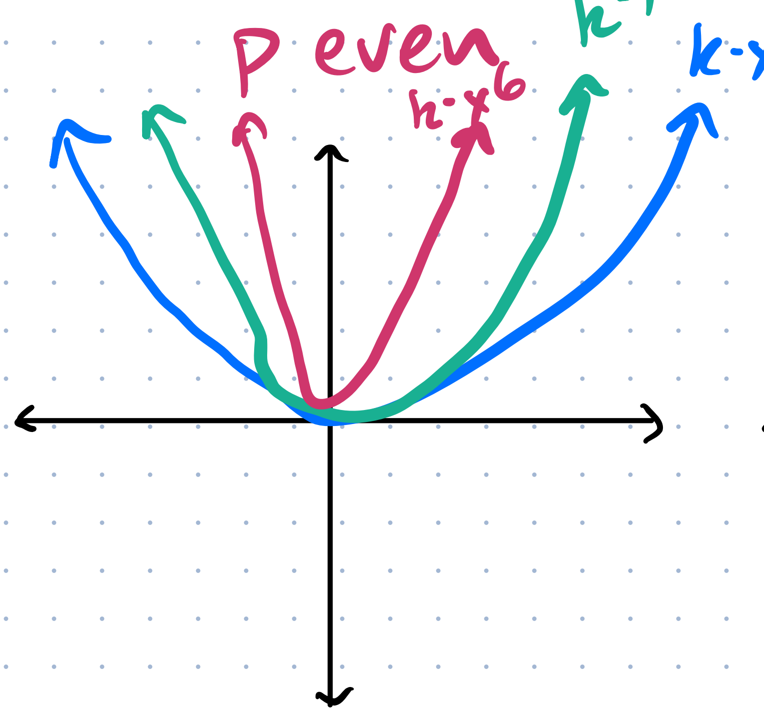
$\cancel{5 \cdot 10^x}$

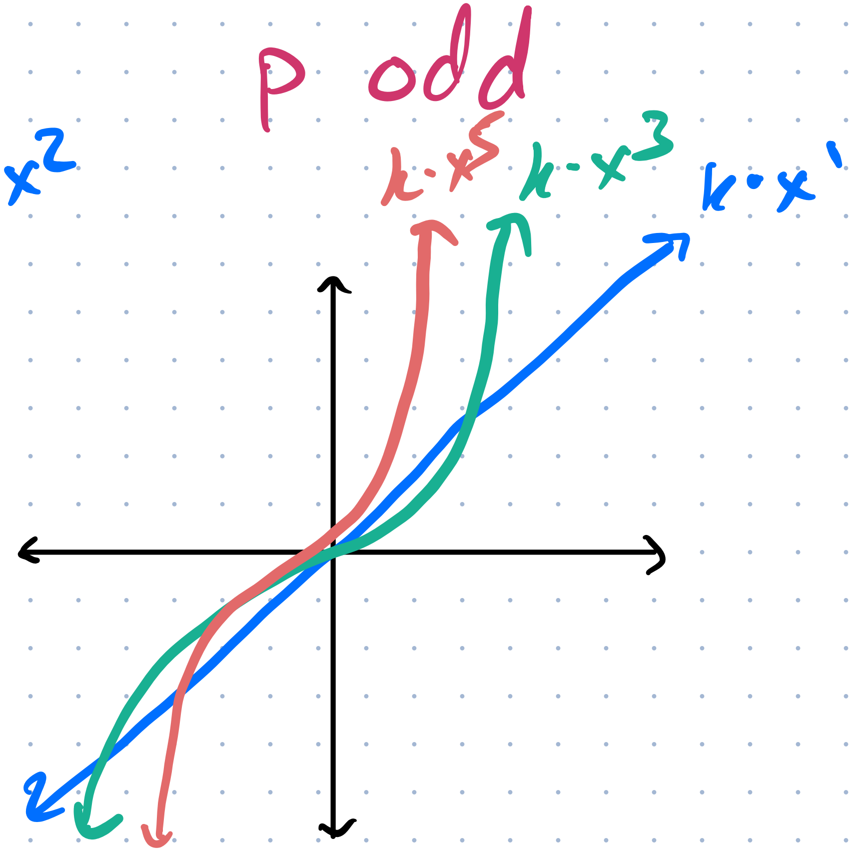
$\frac{1}{2}x^{-3}$

$-5x^{10}$

How to graph kx^p (assuming p is pos. whole #)

If $k>0$:



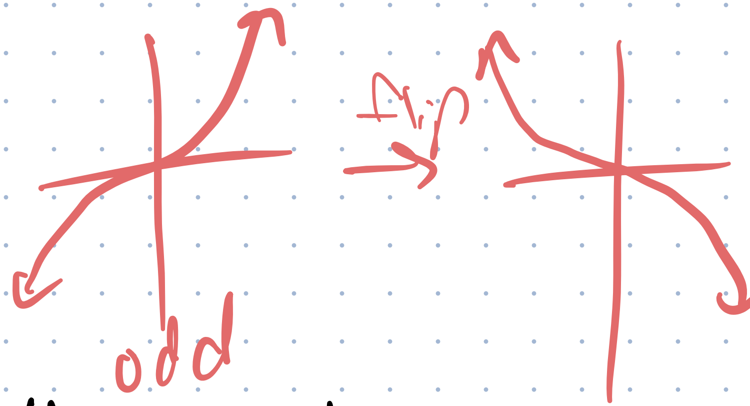


If $k<0$, flip upside down.

How do we know this?

transformation rules

Example: $f(x) = -3x^7$



Q: What is the end behavior of the graph as we go off to the left and right?

Rephrased: What is the "limit" of $f(x)$ as $x \to \infty$?

What is the "limit" of $f(x)$ as $x \to -\infty$?

Example: $f(x) = -3x^7$

$f(\text{big positive \#}) = -3 \cdot (\text{BPN})^7$

$= -3 \cdot (\text{EBPN})$

$= \text{big negative \#}$

limit as $x \to \infty$ is $-\infty$

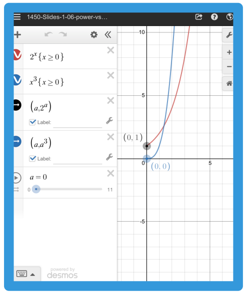
$f(\text{big negative \#}) = ?$

Power functions vs. Exponential functions

$k \cdot x^p$

$k \cdot a^x$

get way bigger
(eventually)



1450-Slides-1-06-power-vs…

$2^x \{x \geq 0\}$

$x^3 \{x \geq 0\}$

$(a, 2^a)$

☑ Label:

(a, a^3)

☑ Label:

$a = 0$

0 11

Polynomials

A polynomial is a bunch of power functions added together.

(as long as the exponents are non-negative whole #s)

degree 4

powers

$$\text{Ex: } p(x) = \underbrace{-3}x^{\boxed{4}} + \underbrace{5}x^{2} \underbrace{-\,2}x^{1} + \underbrace{1}x^{0}$$

constants / coefficients

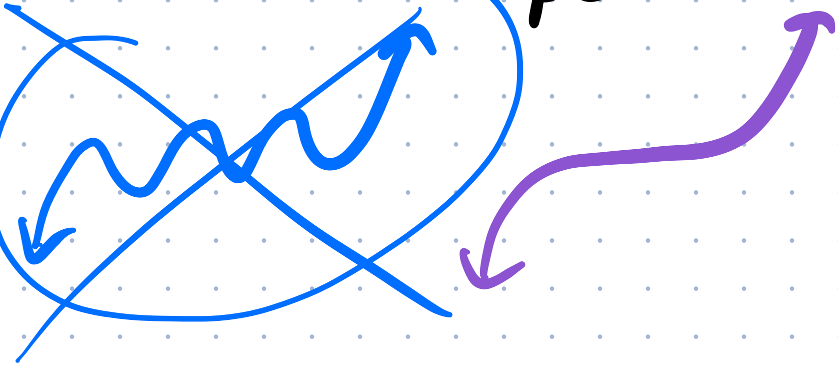
$+0x^3$

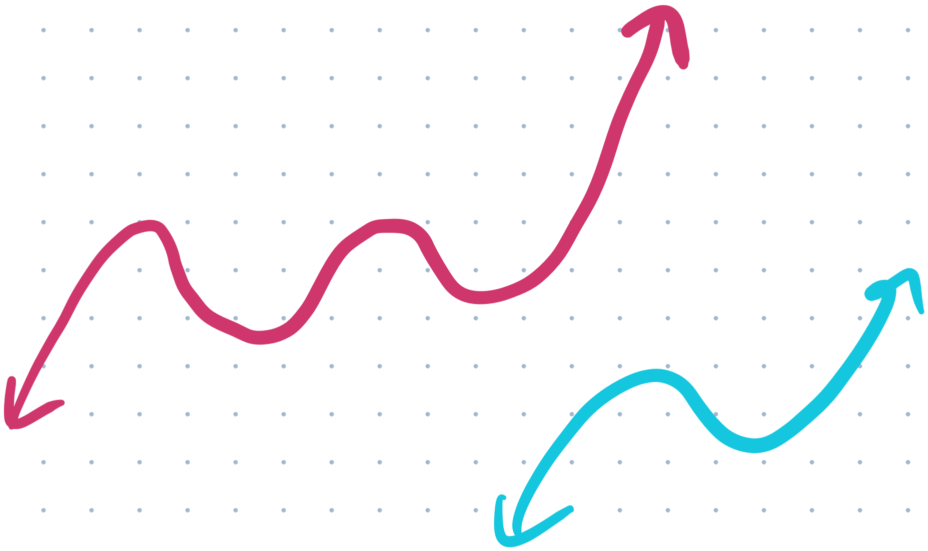
General Form:

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x^1 + a_0$$

degree n

General Shape: n=5



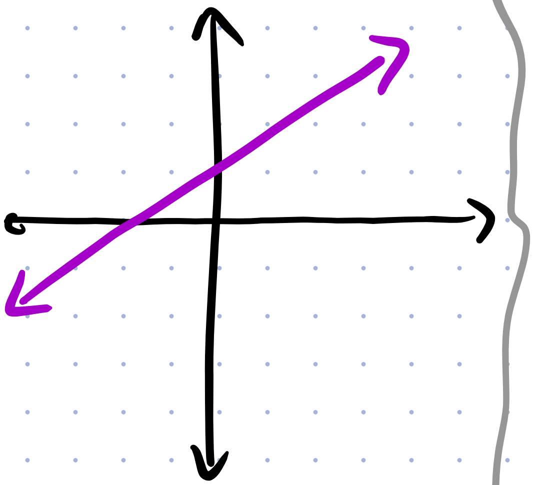


A degree n polynomial can turn around up to n-1 times.

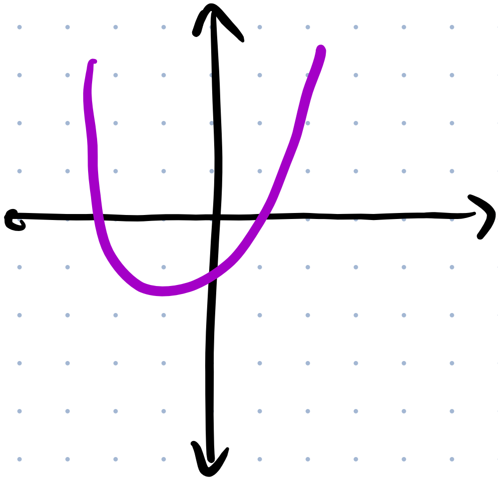
$mx+b$

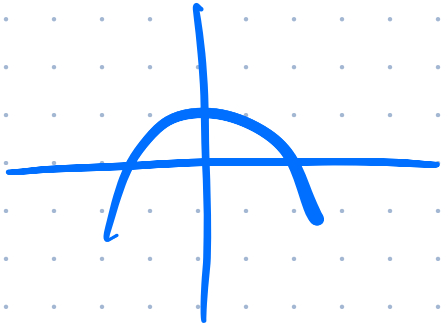
a_1x+a_0

$n=1$

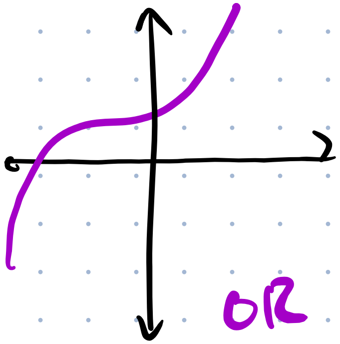


$n=2$





$n=3$



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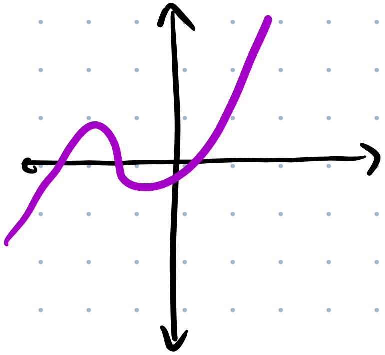
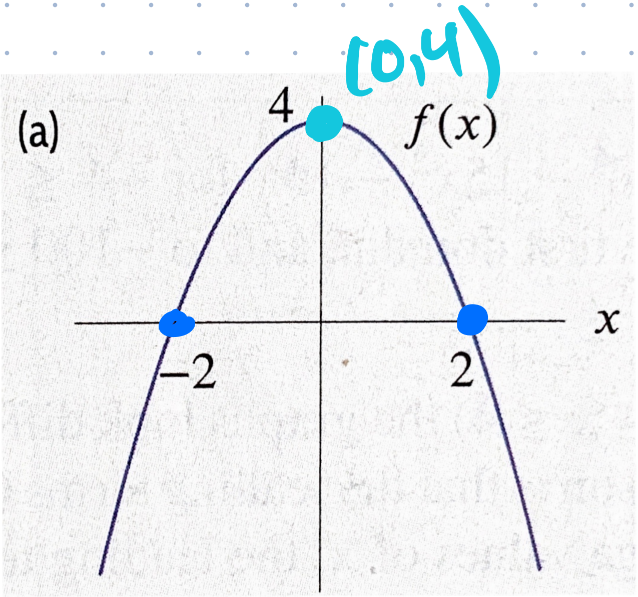


or flips

X-intercepts

If a polynomial $p(x)$ touches or crosses the x-axis at a point $x=c$, then $(x-c)$ must be a factor of $p(x)$.



Can we find a degree 2 polynomial that looks like this?

x-intercepts at 2 and −2

factors $(x-2)$ and $(x-(-2))$

$$x-(-2) = (x+2)$$

$$p(x) = k\cdot(x-2)(x+2)$$

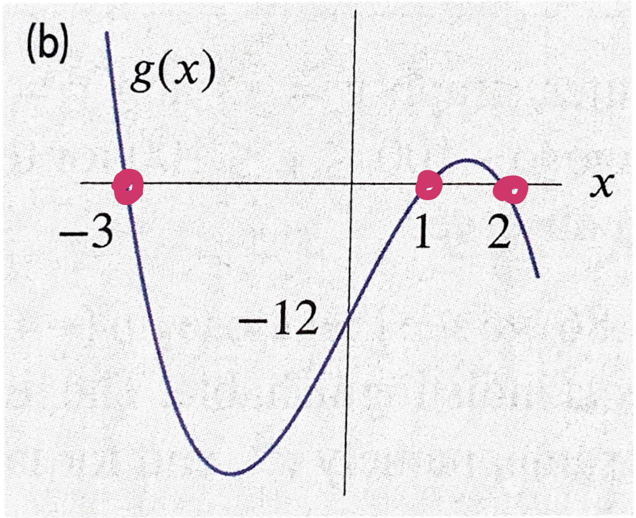
$$4 = p(0) = k\cdot(0-2)\cdot(0+2)$$

$$\Rightarrow \quad 4 = k\cdot(-2)\cdot(+2)$$

$$\Rightarrow \quad 4 = -4k$$

$$\Rightarrow \quad k = -1$$

$$p(x) = -(x+2)(x-2)$$

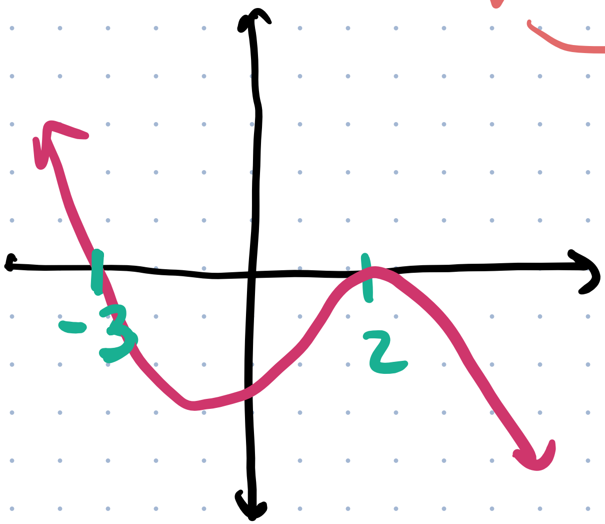


Can we find a cubic (degree 3 poly.) that looks like this?

try this

$$p(x) = -2\cdot(x-1)\cdot(x-2)\cdot(x+3)$$

Note! If the polynomial bounces off the x-axis instead of crossing, then it has an even power of that factor.



$$p(x) = k\cdot(x+3)\cdot(x-2)^{\boxed{2}}$$

Rational Functions

A rational function is just a fraction of polynomials.

$$r(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials

They can be hard to graph, but sometimes not too bad.

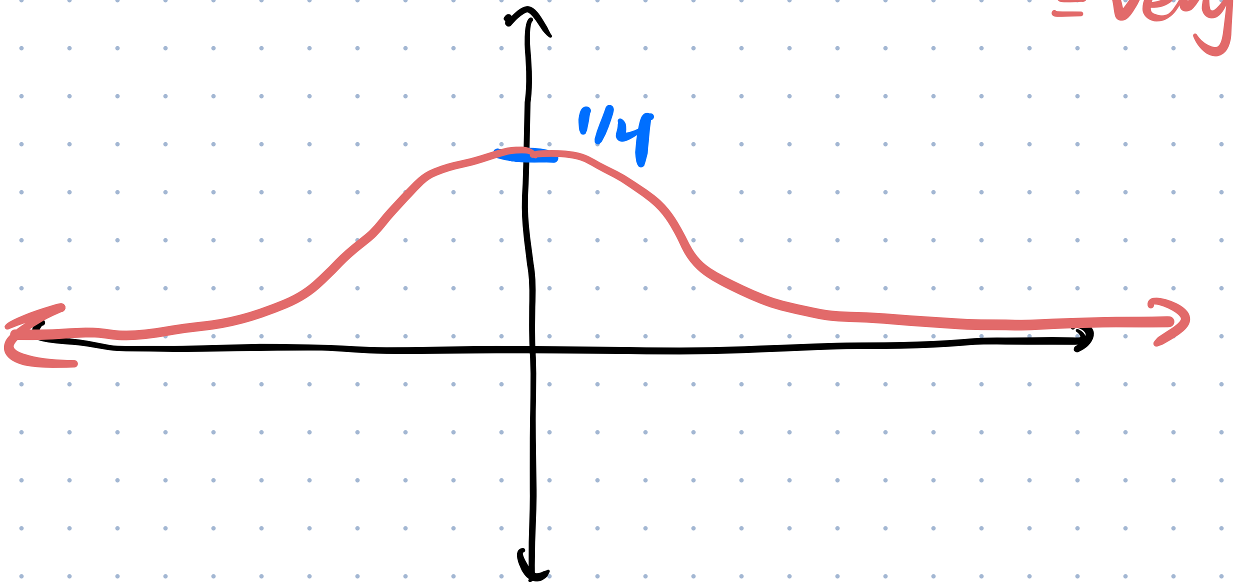
Ex: $f(x) = \dfrac{1}{x^2+4}$ ← degree 0 poly ← degree 2 poly

* $f(0) = \frac{1}{4}$

* $\underline{f(x) \to 0 \text{ as } x \to \infty}$ and as $x \to -\infty$

why? $f(\text{BPN}) = \dfrac{1}{(\text{BPN})^2+4}$

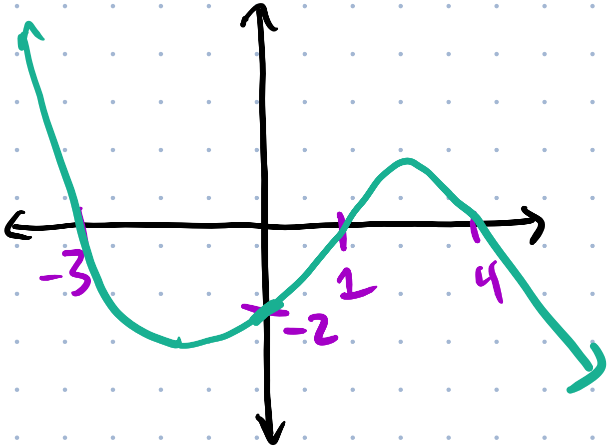
= very small #



The book has many more examples!

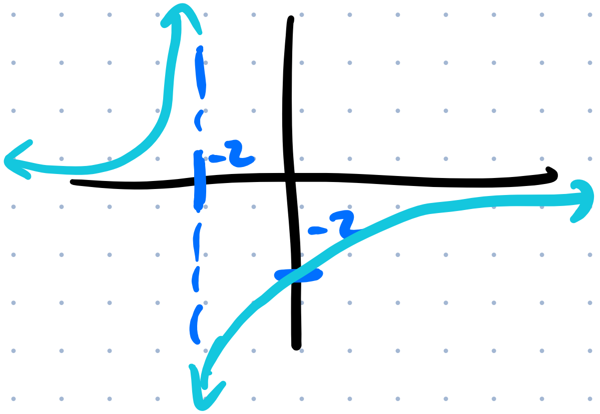
Group work, if time:

(1) Find a cubic polynomial that matches the graph.



$-\frac{1}{6}(x+3)(x-1)(x-4)$

(2) Sketch the rational function $-\frac{4}{x+2}$.



Skipping: Concept of function "domination"

Suggested HW: 1-5, 7-9, 17, 18, 19, 21, 23, 34-36, 41-44, 45-48, 78, 79