Math 1450 - Calculus 1

Wed, Sept. 3

Announcements:

* Calculators - Graphing calc allowed for exams/activities, but you really only need a scientific calculator.
  nothing with wifi capabilities
* First HW due ~~Thurs, Sept 4~~ Sunday, Sept 7, 11:59pm
* First quiz tomorrow! (no calculators for quizzes)
  verbatim from suggested homework
* Course website!
  jaypantone.com → Math 1450

Office Hours

Mondays, 12-1

Wednesdays, 2-3

+ Help Desk!

Today:

→ 1.4: Logarithmic Functions

→ 1.5: Trigonometric Functions

→ 1.6: Powers and Polynomials

The Help Desk is now open!

Math 1450/1455 Help Desk Hours
Fall 2025 (Sep 2 - Dec 5)

Time	Monday	Tuesday	Wednesday	Thursday	Friday
9 - 10 AM	Megan Murphy		Megan Murphy		Navid Mohseni
10 - 11 AM	Brygida Boryczka		Navid Mohseni		Thomas Shomer
11AM - Noon	Brygida Boryczka		Navid Mohseni		Thomas Shomer
Noon - 1PM			Dr. Pantone		Thomas Shomer
1 - 2 PM	Dr. Strifling				
2 - 3 PM	Shahryar Karimi	Megan Murphy	Dr. Spiller	Qishi Zhan	
3 - 4 PM	Dr. Noparstak	Shahryar Karimi	Dr. Noparstak	Qishi Zhan	
4 - 5 PM		Shahryar Karimi		Qishi Zhan	
5 - 6 PM		Sanaz Yousefpanah			
6 - 7 PM		Sanaz Yousefpanah			
7 - 8 PM		Sanaz Yousefpanah			
8 - 9 PM					

The Help Desk is located in the 3rd floor atrium of Cudahy Hall, directly across from the elevators.

You can come to any of these scheduled times.

3rd floor of Cudahy, table near the bathrooms.

Section 1.4 - Logarithmic Functions

* Logarithmic functions are the inverses of exponential functions.

this means they are the reverse operation, like $\sqrt{x}$ is the reverse of x^2.

$$\xrightarrow{a} \boxed{f(x)=x^2} \xrightarrow{a^2} \boxed{g(x)=\sqrt{x}} \xrightarrow{a}$$

(assuming $a \geq 0$)

$$\xrightarrow{a} \boxed{f(x)=2^x} \xrightarrow{2^a} \boxed{g(x)=\log_2(x)} \xrightarrow{a}$$

So, log "undoes" exponentiation.

Ex: Solve $5 = 2^x$.

Take $\log_2$ of both sides

$$\log_2(5) = \underbrace{\cancel{\log_2}(\cancel{2}^x)}$$

$$= x$$

$$x = \log_2(5)$$

$$\approx 2.3219$$

means "the # such that if you raise 2 to it, you get 5"

What two whole #s is the answer between?

$$2^0 = 1$$
$$2^1 = 2$$
$$2^2 = 4$$
$$2^3 = 8$$

x will be between 2 and 3

When the base is 10, it's common to write just "log" instead of "$\log_{10}$".

When the base is e $\approx 2.71\ldots$, we write "ln" and say "natural log".

"ln" = "$\log_e$"

Change of Base Formula:

You can rewrite $\log_a(x)$ as $\dfrac{\log_c(x)}{\log_c(a)}$ for **any** # c.

Useful if your calculator just has a "$\log_{10}$" button.

$$\log_2(5) = \frac{\log_{10}(5)}{\log_{10}(2)} = \frac{\ln(5)}{\ln(2)} = \frac{\log_{17}(5)}{\log_{17}(2)}$$

Properties of Logarithms - for any base a

(1) $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$

(Why? Do "a" to the power of each side.)

Exponents: $a^{x+y} = a^x \cdot a^y$

$$a^{(\log_a(x\cdot y))} = a^{(\log_a(x) + \log_a(y))}$$

$$\cancel{a^{(\log_a}}{(x\cdot y))} = \left(\cancel{a^{\log_a}}{(x)}\right) \cdot \left(\cancel{a^{\log_a}}{(y)}\right)$$

$$x \cdot y = x \cdot y \quad \checkmark$$

(2) $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

(Why? Do "a" to the power of each side.)

Exponents: $a^{x-y} = \frac{a^x}{a^y}$

You try!

(3) $\log_a(x^y) = y \log_a(x)$

(Why? Do "a" to the power of each side.)

Exponents: $a^{(x \cdot y)} = (a^x)^y$

You try for practice

(4) $\cancel{\log_a}(\cancel{a}^x) = x$ (5) $\cancel{a}^{\cancel{\log_a}(x)} = x$

Because logs are the inverse functions of exponentials

$\log_7(7^3) = 3$

$\cancel{\ln}(\cancel{e}^3) = 3$

$\ln(e^{-3}) = -3$

Group Work (if time):

Simplify $\underbrace{\log_{10}(A^2 B)} - \underbrace{\log_{10}\left(\frac{A}{C}\right)} + \ln\left(\frac{1}{e^2}\right)$ $\quad \frac{1}{e^2} = e^{-2}$

$$\log_{10}(A^2) + \log_{10}(B) - \left(\log_{10}(A) - \log_{10}(C)\right) + \ln(e^{-2})$$

$$\underline{2\log_{10}(A)} + \log_{10}(B) \underline{- \log_{10}(A)} + \log_{10}(C) - 2$$

$$\log_{10}(A) + \log_{10}(B) + \log_{10}(C) - 2$$

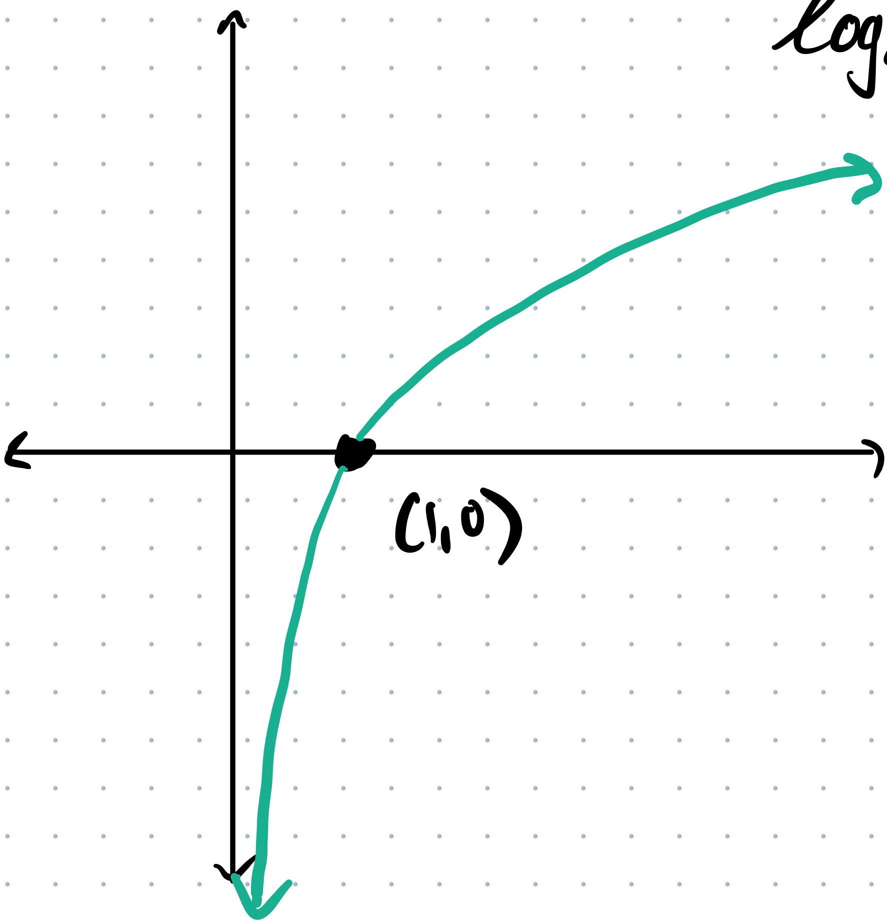
$$= \log_{10}(A \cdot B \cdot C) - 2$$

Shape of a log

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

$\log_a(x)$, if $a > 1$

off to $+\infty$, slowly

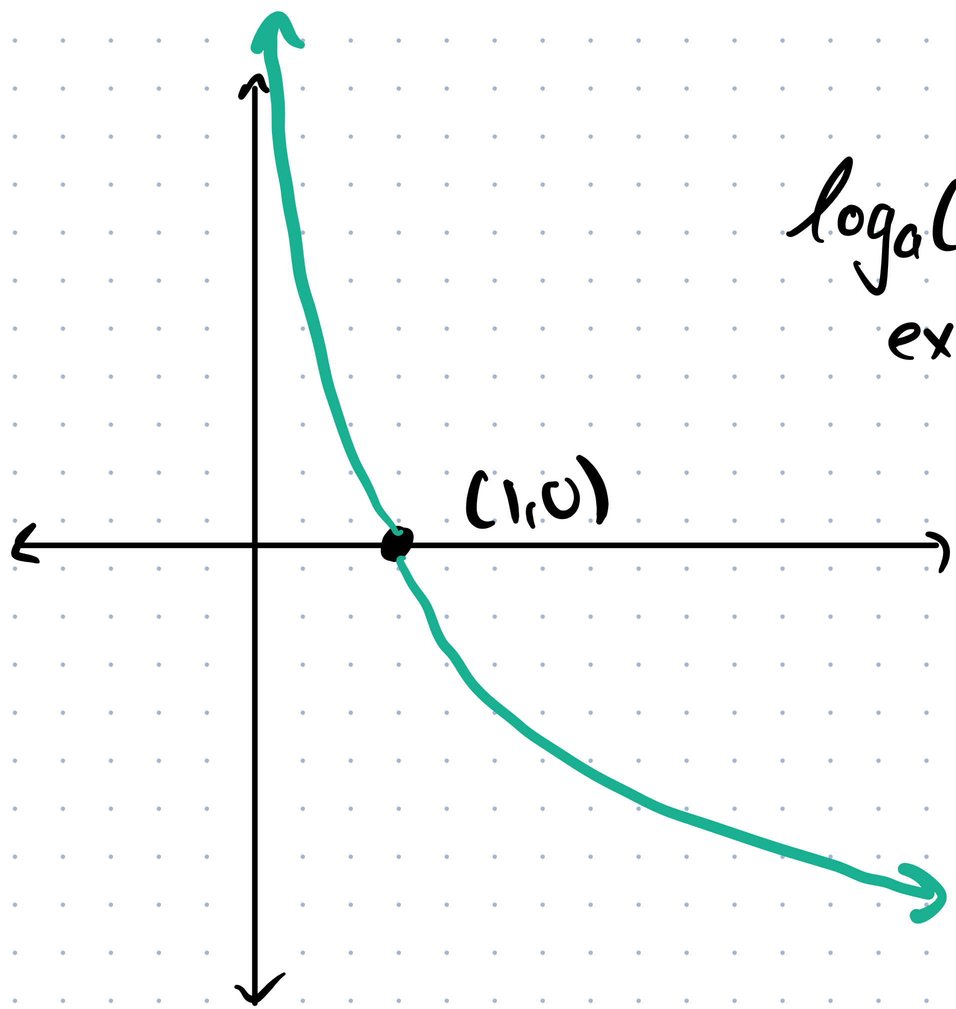


vertical asympt. to $-\infty$
at $x=0$

$\log_a(1) = 0$
why?
$a^0 = 1$

$\log_a(0)$ is undefined
$\log_a(x)$ where $x<0$,
also undef.

Shape of a log



$\log_a(x)$ if $0<a<1$

example: $\log_{1/2}(x)$

Same domain and range as the last one.

Example:

$14.235 \cdot \underline{(0.03)}^t$

Suppose the population of Burkina Faso is estimated by the function $\underline{P(t) = 14.235 \cdot (1.03)^t}$, where t is the number of years after 2007 and $P(t)$ is in millions of people.

When will the population reach 30 million?

Q: For what value of t do we get $P(t) = 30$?

$$14.235 \cdot (1.03)^t = 30$$

$$\Rightarrow (1.03)^t = \frac{30}{14.235}$$

$$\Rightarrow \cancel{\log_{1.03}}\left[(1.03)^t\right] = \log_{1.03}\left(\frac{30}{14.235}\right)$$

$$\Rightarrow t = \log_{1.03}\left(\frac{30}{14.235}\right) = \left[\frac{\log\left(\frac{30}{14.235}\right)}{\log(1.03)}\right] \approx \underline{25.2206}$$

Answer: 2032

<u>Suggested HW</u>: 1, 2, 5, 7, 8, 10, 15, 25, 27, 36, 37, 41, 64, 66

Section 1.5 - Trigonometric Functions

There are two units of measure for angles:
degrees and radians (like feet vs. meters)

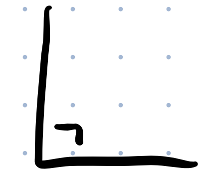
A circle has 360°, or 2π radians, so the conversion factor is

$$(\text{degrees}) = \frac{360}{2\pi} \cdot (\text{radians})$$

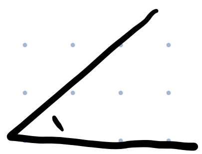
$$= \frac{180}{\pi} \cdot (\text{radians})$$

$$(\text{radians}) = \frac{\pi}{180} \cdot (\text{degrees})$$

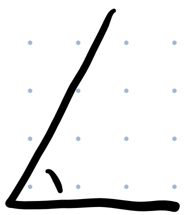
$$\text{deg} = \frac{180}{\pi} \cdot \text{rad} \qquad \text{rad} = \frac{\pi}{180} \cdot \text{deg}$$



$90° = \frac{\pi}{2}$ radians



$45° = \frac{\pi}{4}$ radians

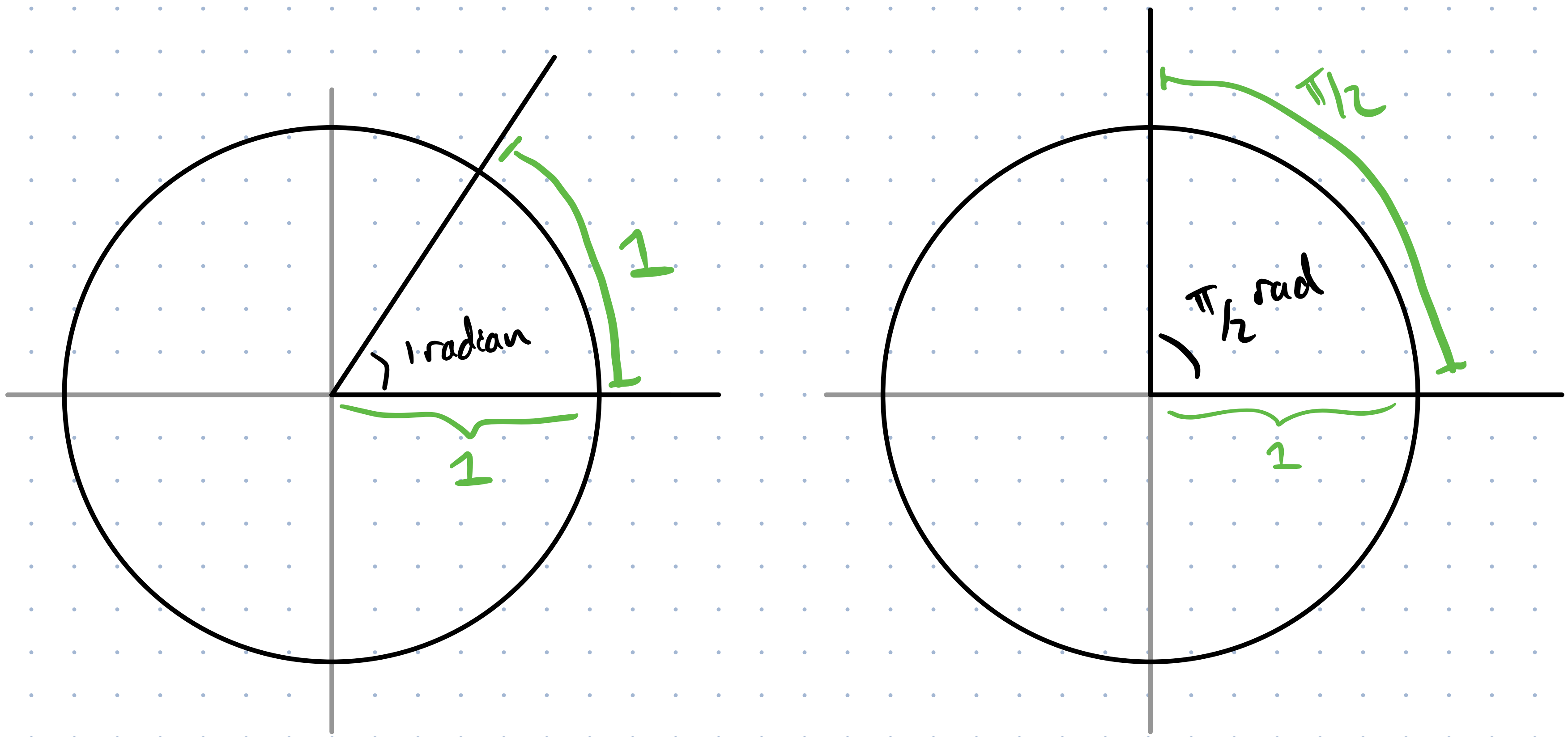


1 rad = ≈ 57.30 degrees

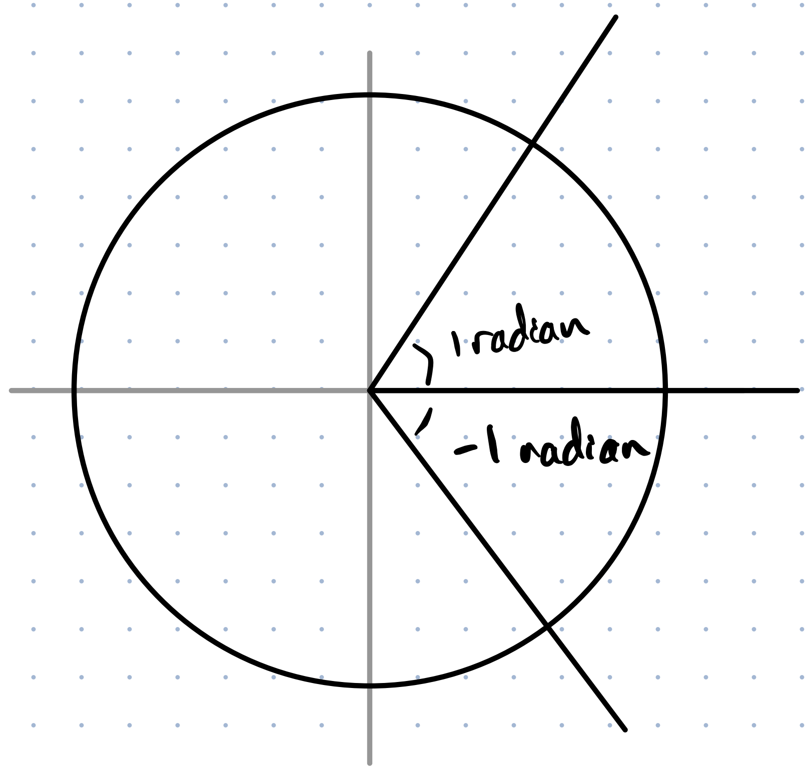
$1 \text{ radian} \cdot \frac{180}{\pi} = \frac{180}{\pi}$ degrees

The point of radians is that they correspond to arc length for a unit circle (radius = 1)

full circle = 2π radians = 2π circumference

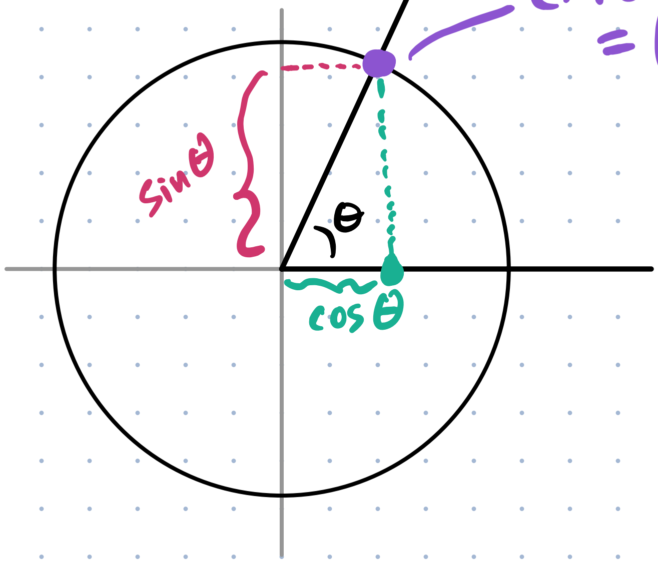


Angles technically have a direction. Positive angles go counterclockwise, negative angles go clockwise.



Sine and cosine

As you spin a point around the outside of a circle, the trig functions cos and sin tell you how the x and y coordinates of that point change.



$$(x, y) = (\underbrace{\cos(\theta)}_{\text{x-coord}}, \underbrace{\sin(\theta)}_{\text{y-coord}})$$