Math 1450 – Calculus 1

Fri, Aug 29

Announcements:

* No class on Monday
* Calculators – Graphing calc allowed for exams/activities, but you really only need a scientific calculator.
  nothing with wifi capabilities
* First HW due Thurs, Sept 4 – Wiley Plus
* First quiz same day (no calculators for quizzes)
* Course website!
  jaypantone.com → Math 1450

Today:

→ 1.2: Exponential Functions
→ 1.3: New Functions from Old
→ 1.4: Logarithmic Functions

Office Hours

Mondays, 12–1

Wednesdays, 2–3

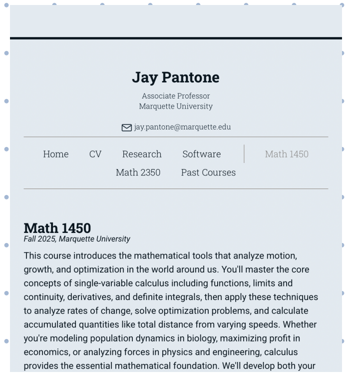
+ Help Desk!

Old Lecture/Exercise Videos

→ In Fall 2020 this class was all pre-recorded videos.

→ For each section I recorded 2 videos:

- * A lecture on the material
- * A video working through 5 exercises



→ I'm putting links to them on the course calendar

→ The material is close but not the same, and some of the sections are different, so it's not a substitute for class!

→ But they might help if you're stuck on a topic.

Exponential Growth Formula:

$$P(t) = \underbrace{14.235}_{\text{starting value / y-intercept}} \cdot \underbrace{(1.03)}_{\text{base}}^{t}$$

independent variable

base

starting value
y-intercept

"growth rate"
= base − 1
1.03 − 1 = 0.03

The general form for exponential growth is

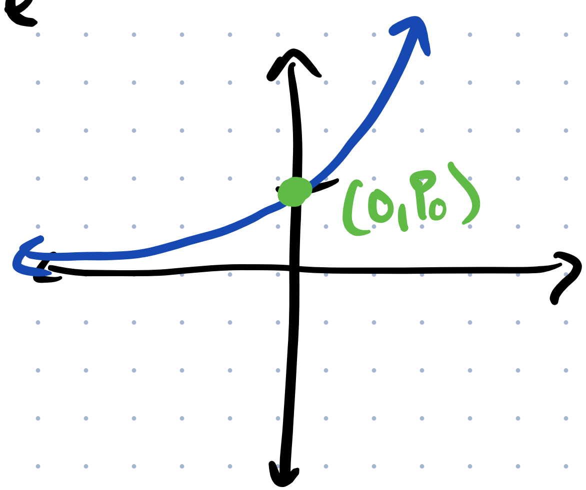
$$P(t) = P_0 \cdot a^t$$

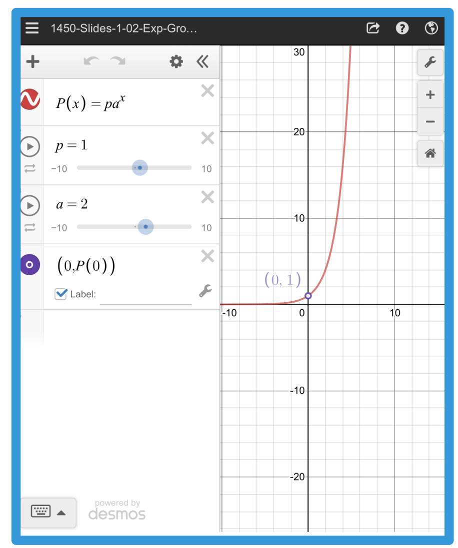
base

growth rate $= a - 1$

For $P(t)$ to <u>grow</u>, we need $a > 1$ and $P_0 > 0$.

General shape:

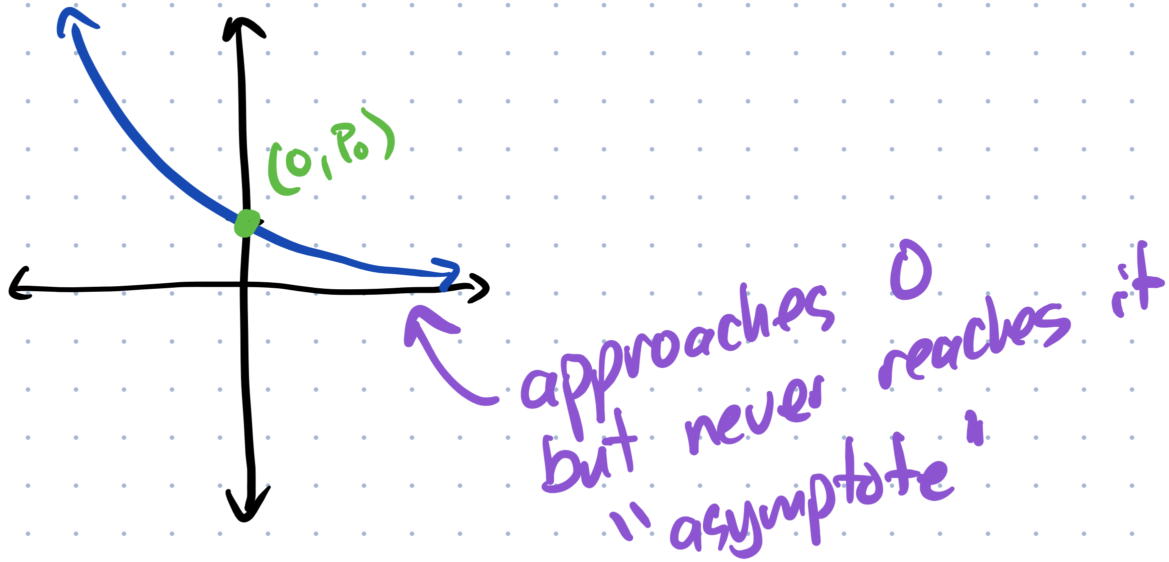




Exponential Decay

$P(t) = P_0 a^t$

When $0 < a < 1$, we have exponential decay



Example:

Your body filters medication from your blood at a rate that depends on the medication. Ampicillin is filtered at a rate of 60% per hour.

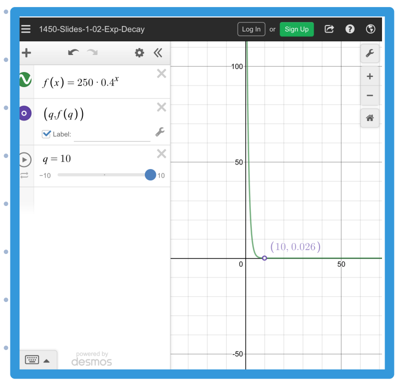
60% gone = 40% is left

Suppose you start with 250 mg in your blood, and let $f(t)$ be the function for the amount left after t hours.

$$f(0) = 250 \cdot (0.4)^0$$
$$= 250 \cdot 1 = 250 \checkmark$$

$$f(t) = 250 \cdot (0.4)^t$$

growth rate $= -0.6$



General Exponential Functions

We say P is an **exponential function** of t with base a if

$$P = P_0 a^t,$$

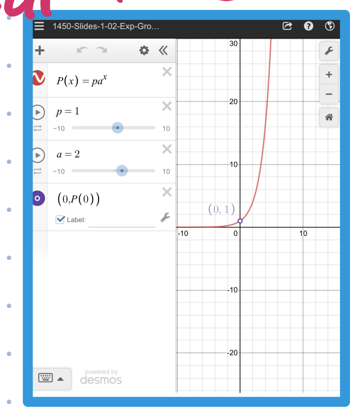
where P_0 is the initial quantity (when $t = 0$) and a is the factor by which P changes when t increases by 1.
If $a > 1$, we have exponential growth; if $0 < a < 1$, we have exponential decay.

$a > 1$: exponential growth

$0 < a < 1$: exponential decay

$a = 1$: $P = P_0 \cdot (1)^t \Rightarrow P = P_0$, horizontal line

$a < 0$: doesn't make sense



Some terminology:

"Doubling Rate": the time it takes an exponentially growing quantity to double

"Half-life": the time it takes an exponentially decaying quantity to reduce by 50%.

250 mg $\rightarrow$ 125 mg

Base e

"e" is a predefined #, kind of like π

$e \approx 2.71828\ldots$ (goes on forever without repeating)

For reasons we'll see in a later chapter, "e" is a very convenient quantity for the exponential growth rate.

Plus, you can always rewrite $P_0 a^t$ ← general form

to use "e" — with a little bit of cheating.

Ex:

$$5\cdot 2^t = 5\cdot e^{\ln(2)\cdot t}$$

$$\approx 5\cdot e^{0.693t}$$

$$a^{x\cdot y} = (a^x)^y$$

Why? Rules of exponents

$$e^{\ln(2)\cdot t} = \left(\cancel{e}^{\cancel{\ln}(2)}\right)^t = 2^t$$

For decay: $5 \cdot \left(\frac{1}{3}\right)^t = 5 \cdot e^{\ln\left(\frac{1}{3}\right)t}$

$\approx 5 \cdot e^{-1.099 \cdot t}$

When we rewrite with a base of "e", we call the constant like 0.693 or −1.099 the "continuous rate"

When the base is e, we call the constant like 0.693 or -1.099 the "continuous rate".

Topics in 1.2 we didn't cover:

- Concavity

Suggested HW

1.2: # 6, 7, 11, 13, 15, 25, 27, 29, 37, 41, 44, 65

Section 1.3 - New Functions From Old

Transformations to a function $f(x)$:

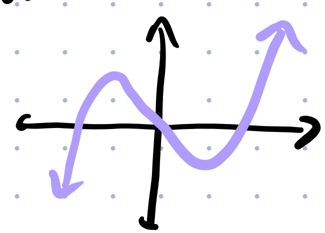
$f(x) \pm c$ $f(x)+5$

$f(x \pm c)$ $f(x+5)$

$c\,f(x)$

$f(cx)$

Example function: $f(x) = x^3 - 3x$



$f(x) \pm c$

$\boxed{f(x) = x^3 - 3x}$

Adding a positive # c shifts <u>vertically up</u> by c units.
Adding a negative # c shifts <u>vertically down</u> by c units.

Why? A point on $f(x)$ is $f(2) = 2$.

$f(2) = 2^3 - 3 \cdot 2$
$= 8 - 6 = 2$ ✓

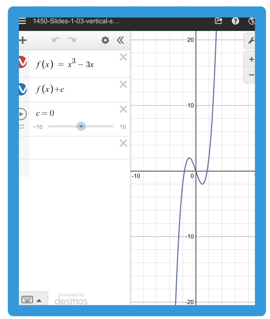
Let $g(x) = f(x) + 3$.

$g(2) = f(2) + 3 = 2 + 3 = 5$

Let $h(x) = f(x) - 7$.

$h(2) = f(2) - 7 = 2 - 7 = -5$

$2 \rightarrow \boxed{f} \rightarrow 2 + 3 = 5$



$f(x \pm c)$

Adding a positive # c inside shifts to the left by c units.
Adding a negative # c inside shifts to the right by c units.

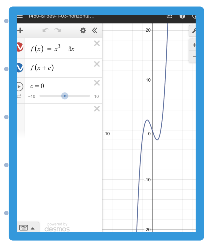
Why? Consider $g(x) = f(x+5)$.

The value of g at $x=0$ is the value of f at $x=5$.

$5^3 - 3 \cdot 5 = 110$

$g(0) = f(0+5) = f(5)$
$= 110$

The point $(5, 110)$ is on f and so the point $(0, 110)$ is on g.



g pulls its values from 5 to the right, so those values go 5 to the left.

$c\,f(x)$

Multiplying by c causes a vertical <u>stretch</u> or <u>shrink</u>.

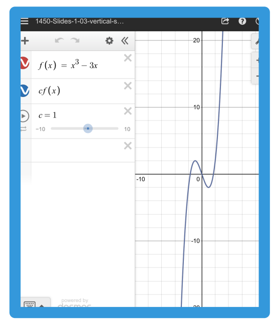
$5 \cdot f(x)$	$c > 1$:	vertical stretch
$1 \cdot f(x)$	$c = 1$:	no change
$\frac{1}{2} \cdot f(x)$	$0 < c < 1$:	vertical shrink
$0 \cdot f(x)$	$c = 0$:	function becomes 0
$-\frac{1}{2} \cdot f(x)$	$-1 < c < 0$:	vertical shrink and flip
$-f(x)$	$c = -1$:	just vertical flip
$-5\,f(x)$	$c < -1$:	vertical flip and stretch

$c\,f(x)$

Multiplying by c causes a vertical <u>stretch</u> or <u>shrink</u>.

Why? If $g(x) = 5f(x)$ then the point $(2, f(2)) = \underline{(2,2)}$ becomes

$g(2) = 5 \cdot f(2) = 10.$



If $h(x) = -\frac{1}{2}f(x)$ then the point $(2,2)$ becomes

$f(cx)$

$f(x) = x^3 - 3x$

$f(2) = 2$

$c > 1$: horizontal shrink

$0 < c < 1$: horizontal stretch

$-1 < c < 0$: horizontal stretch + flip

$c < -1$: horizontal shrink + flip

Why? Let $g(x) = f(5x)$.
Then g "grabs" its values from further away.

$f(10) = 10^3 - 3 \cdot 10 = 970$

$g(2) = f(5 \cdot 2) = f(10) = 970$

So the point $(10, 970)$ moves to $(2, 970)$.

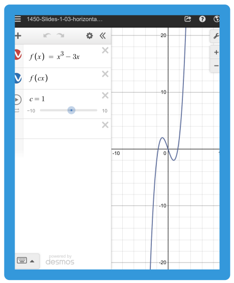
$f(cx)$

Let $h(x) = f(\frac{1}{2}x)$. Then h "grabs" its points from closer to the axes.

$(2, f(2))$ moves to $(2, g(2))$

$= (2, f(\frac{1}{2} \cdot 2))$

$= (2, f(1))$



Recap:

$f(x) \pm c$: vert. shift

$f(x \pm c)$: hor. shift

$c\,f(x)$: vert. stretch/shrink/flip

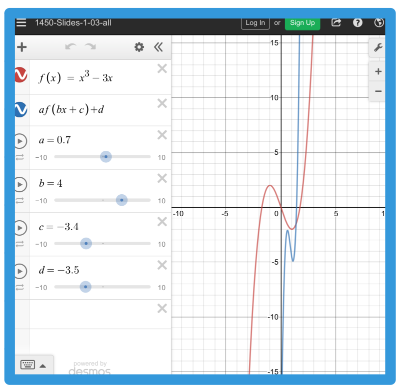
$f(cx)$ hor. stretch/shrink/flip

inside = horizontal

outside = vertical

inside = opposite

And you can combine these!



Composition of functions

Composing two functions means doing one first, then the other

Ex: $f(x) = x^2$, $g(x) = x - 2$

"square" "subtract 2"

$$f(g(x)) = f(x-2) = (x-2)^2 = x^2 - 4x + 4$$

$$\underset{3}{x} \rightarrow \boxed{g} \xrightarrow[1]{x-2} \boxed{f} \xrightarrow[1]{(x-2)^2}$$

$f(g(3)) = 1$

not the same!

$$g(f(x)) = g(x^2) = x^2 - 2$$

$$x \rightarrow \boxed{f} \xrightarrow{x^2} \boxed{g} \rightarrow x^2 - 2$$

$g(f(3)) = 7$

Order matters! Inside to outside.

$3 \longrightarrow 9 \longrightarrow 7$

Example: Let $f(x) = x^2+2$ and $g(x) = \sqrt{x}-5$.
Compute $f(g(x))$ and $g(f(x))$?

$$f(g(x)) = f(\sqrt{x}-5) = (\sqrt{x}-5)^2+2$$
$$= x - 10\sqrt{x} + 27$$

$$g(f(x)) =$$
$$g(x^2+2) = \sqrt{x^2+2} - 5$$

different

Skipping for now: Inverse functions
Odd/even functions

Suggested HW:

1.3: #1-5, 9-12, 15-17, 41-46, 47-49