As a reminder, Exam 3 will be held in class on **Wednesday November 12**. The exam will cover Sections 3.5-3.7, 3.9, 4.1-4.3, and 4.6. A list of topics is included below.

The discussion session before the exam, Tuesday November 11, is scheduled to be a review day. As a guide for this discussion, there is a list of recommended problems out of the textbook included after the topics list below.

To access the review exercises, you'll need to log in to Wiley and go to the tab marked "Wiley Course Resources". Then, for the Chapter 3 exercises, click the down arrow next to "Ch 3: Short-Cuts to Differentiation (45)". The second item should be "Review Material and Projects: Chapter 03". Clicking this will bring up the review exercises as a downloadable pdf. For the Chapter 4 exercises, the process is similar.

The answers to all of the review questions will be posted in a separate pdf.

Of course any other material you wish to study is also a good idea (old homework, quizzes, textbook, lecture notes, etc.).

Suggested review problems from the textbook

Ch 3 Rev Ex's # 9, 11, 13, 14, 17-23, 27-30, 38, 40, 49, 50, 55, 68, 76-80, 96-97(f only), 106, 107

Ch 4 Rev Ex's # 1-22, 24, 27, 30, 31, 34-36, 39-42, 48-52, 54, 65-72, 75, 81, 89, 90, 92, 94-99

Exam 3 Review Topics for MA 1450

Chapter 3: First Uses of the Derivative

- Derivatives of trigonometric functions (§3.5)
  * $\frac{d}{dx}\big[\sin(x)\big] = \cos(x)$
  * $\frac{d}{dx}\big[\cos(x)\big] = -\sin(x)$
  * $\frac{d}{dx}\big[\tan(x)\big] = \frac{1}{\cos^2(x)} = \sec^2(x)$
- Derivatives of inverse functions (§3.6)
  * $\frac{d}{dx}\big[\ln(x)\big] = \frac{1}{x}$
  * $\frac{d}{dx}\big[\arcsin(x)\big] = \frac{1}{\sqrt{1-x^2}}$
  * $\frac{d}{dx}\big[\arctan(x)\big] = \frac{1}{1+x^2}$
  * $\frac{d}{dx}\big[f^{-1}(x)\big] = \frac{1}{f'\big(f^{-1}(x)\big)}$

- Implicit differentiation and slopes of implicit curves (§3.7)
- Linear approximation of a differentiable function (§3.9)

  $$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

- Use a linear approximation to estimate values (§3.9)

Chapter 4: Applications of the Derivative

- Find the critical points of a function (§4.1)
- Classify the critical points of a function as local maxima, local minima, or neither (§4.1)
  - First Derivative test
  - Second Derivative test
- Find the inflection points of a function (§4.1)
- Use sign charts for f' and f'' to graph f (§4.1)
- Find the global max/min of f on a closed interval $[a, b]$ (§4.2)
- Find the global max/min of f on a non-closed interval (§4.2)
- Optimization word problems (§4.3)
- Related rate word problems (§4.6)

Additional Review Problems

1. Find the derivative of the function.

(a) $y = 4\sin(t) - 10\tan(t)$

(b) $f(x) = \dfrac{8}{x^4} - \dfrac{\cos(4x)}{8}$

(c) $y = 6\tan^3(\theta)$

(d) $P = \dfrac{1 + 5\tan(r)}{1 - 5\tan(r)}$

(e) $y = x^7\cos(4x)$

(f) $y = \sin(\cos(7t))$

(g) $h(x) = 10e^{2\sin(\pi x)}$

(h) $\ell(t) = \sqrt[3]{\sin^5(6t)}$

(i) $s = \cos(t^2 e^t)$

(j) $y = \theta^{1.2} - 0.5e^{\tan(\theta^3)}$

2. Find the second derivative of $f(x) = \cos(\pi x^2)$.

3. Find an equation of the tangent line of $f(\theta) = \tan(2\theta)$ at $\theta = \dfrac{\pi}{8}$. Use exact answers.

4. Assume $g(\pi) = 0$ and $g'(\pi) = -2$.

(a) $G'(\pi)$ when $G(x) = \sin(g(x))$

(b) $H'(\pi)$ when $H(x) = \dfrac{g(x) + 2}{\cos(x) + 2}$.

5. Differentiate the function.

(a) $y = 2\ln(x^{10}) + \dfrac{2}{x^{10}} + 10$

(b) $y = x^3\ln(9x)$

(c) $f(x) = \ln(x^4 + 8)$

(d) $g(t) = \dfrac{5t + 1}{\arcsin(t)}$

(e) $y = 3\arctan(x^9)$

(f) $y = \sin(4x)\arcsin(8x)$

(g) $p = \cos(\ln(1 + e^r))$

(h) $y = \sqrt{1 + \arctan(7t)}$

(i) $v(x) = 1 - 0.5e^{5\arcsin(x^2)}$

(j) $f(x) = \ln(2x + e^{3x})$

(k) $f(x) = \ln(2xe^{3x})$

(l) $s = \arctan\left(\dfrac{1}{2x + 1}\right)$

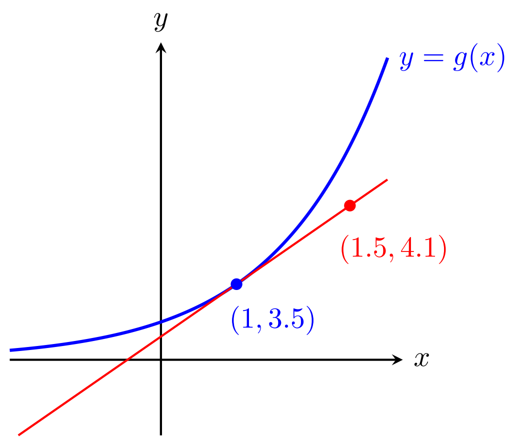
6. Find the second derivative of the function $f(x) = \arcsin(3x)$.

7. Let $f(x) = 3 + \ln(x - 1)$.

(a) Sketch a graph of the function $f(x) = 3 + \ln(x - 1)$ with the graph of its tangent line at $x = 2$. Find the slope of the tangent line.

(b) Find an equation of the tangent line of $f(x) = 3 + \ln(x - 1)$ at $x = 2$.

8. Below is the graph of the $y = g(x)$. Find the derivative $G'(1.0)$ when $G(x) = \ln(g(x))$.



9. Use implicit differentiation to find $\frac{dy}{dx}$.

(a) $\frac{1}{x^2} + y^2 = 1$

(b) $xy + 4x^2 = e^{5y}$

(c) $x^4y^8 - \ln(x) = \ln(y) + 1$

(d) $\arctan(7x) + \arctan(7y) = y^2$

(e) $\frac{x^6}{y^3} = 4y - 1$

(f) $\sin(xy) = x^2y^2$

(g) $x^2\cos(y) = e^{x^2+y^2}$

10. (a) Use implicit differentiation to find $\frac{dy}{dx}$ for the equation $x^4 + x^2y - y^2 = -1$.

(b) Use Part (a) to find the slope of the tangent line of $x^4 + x^2y - y^2 = -1$ at the point $(1, 2)$.

(c) Find an equation of the tangent line in Part (b).

11. Sketch a graph of the function $f(x) = 2 + \sqrt[3]{x}$ and its tangent line at $x = 1$

(a) Find the tangent line approximation of $f(x) = 2 + \sqrt[3]{x}$ near $x = 1$.

(b) Use the tangent line approximation to approximation $2 + \sqrt[3]{1.06}$.

(c) From looking at the graph in Part (a), is the approximation of $2 + \sqrt[3]{1.06}$ an over approximation or under approximation. Explain your reasoning.

12. Use the local linearization of $f(x) = \frac{1}{\sqrt{x}}$ near $x = 16$ to approximate $\frac{1}{\sqrt{15.5}}$.

13. Find the tangent line approximation for $f(x) = e^{-x^2}$ at $x = -1$.

14. (a) Find the local linearization of $f(x) = 2\ln(x)$ near $x = 1$.

(b) The equation $2\ln(x) = 0.2 - 0.5x$ has a solution near $x = 1$. Use Part (a) to find an approximate value for the solution.

15. Assume the function $y = g(x)$ satisfies $g(3) = 100$ and $g'(3) = -4.25$

(a) Find the tangent line approximation of $y = g(x)$ near $x = 3$.

(b) Use the tangent line approximation in Part (a) to approximate $g(3.5)$.

(c) Assuming $g''(3) = -2$, is the approximation of $g(3.5)$ likely an over estimation or an under estimation? Explain your reasoning. [Hint. Think about concavity of $y = g(x)$ at $x = 3$.]

16. Find all the critical points of the function.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 5$

(b) $f(x) = (x^2 - 1)^3$

(c) $y = x^2 + \dfrac{1}{x^2}$

(d) $g(x) = xe^{4x}$

(e) $y = xe^{bx}$ where $b > 0$ is a constant

(f) $y = \dfrac{x}{12} - \sqrt[3]{x}$

17. (a) Explain why $x = 0$ is a critical point of $f(x) = \sqrt[3]{x}$.

(b) Explain why $x = 0$ is *not* a critical point of $g(x) = \dfrac{1}{x^3}$.

18. For the given functions, answer the following questions *without* graphing the function.

i) Find all the critical points of f;

ii) Find the intervals of increasing/decreasing using the sign of the first derivative f';

iii) Use the First Derivative Test to classify the critical points as local maximum/minimum, or neither;

iv) Find the intervals of concave up/concave using the sign of the second derivative f'';

v) Find the inflection points of f.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 5$

(b) $f(x) = (x^2 - 1)^3$

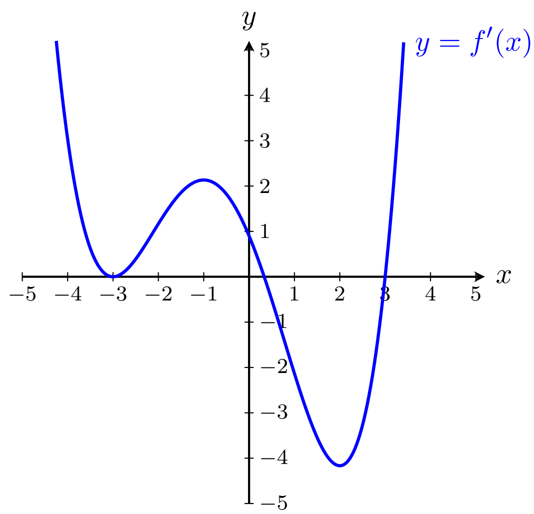
(c) $f(x) = xe^{4x}$

(d) $f(x) = x^2 + \dfrac{1}{x^2}$

19. The graph below is the graph of the <u>*derivative*</u> $f'(x)$ of $f(x)$.

(a) Find all the critical points of the function $y = f(x)$. Careful! Below is graph of its derivative $y' = f'(x)$.

(b) Use the First Derivative Test to classify the critical points of the function $y = f(x)$ as local maximum, local minimum, or neither.



20. Use the Second Derivative Test to classify the critical points of the function as local maximum or local minimum.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 5$

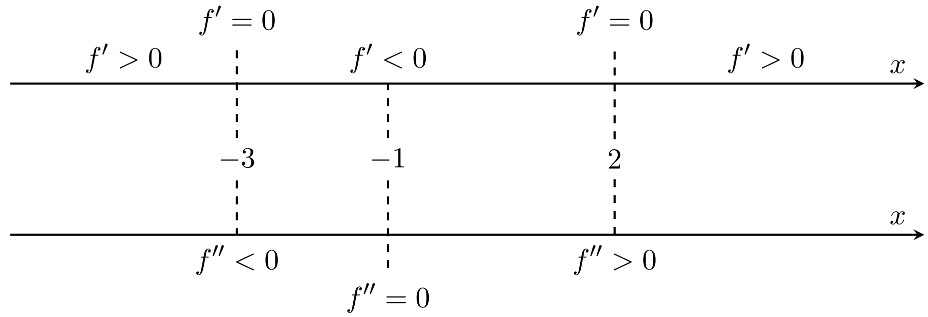
(b) $f(x) = (x^2 - 9)^2$

(c) $y = x^2 + \dfrac{1}{x^2}$

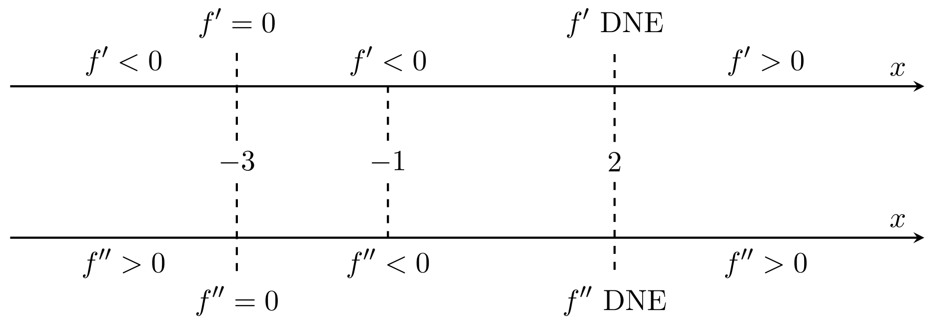
(d) $y = xe^{bx}$ where $b > 0$ is a constant

21. Sketch a possible graph of the function $y = f(x)$ using the given information about the derivatives $y' = f'(x)$ and $y'' = f''(x)$. Label local extrema and points of inflection.

(a)



(b)



22. Use the calculus techniques established in Section 4.2 to find the global maximum and global minimum of the function on the indicated interval.

(a) $f(x) = x^4 - 8x^2 + 10$ on the interval $-1 \le x \le 5$

(b) $f(x) = (x^2 - 1)^3$ on the interval $-3 \le x \le 0$

(c) $f(x) = x^2 e^x$ on the interval $[-3, 3]$

(d) $f(x) = \dfrac{x}{3} - \sqrt[3]{x}$ on the interval $[-1, 8]$

23. (a) Use calculus techniques established in Section 4.2 to find the global minimum of the function $g(x) = \dfrac{1}{x^3} - \dfrac{1}{x^2}$ over the interval $0 < x < \infty$. Explain how you arrived at your answer.

(b) Does the function $g(x) = \dfrac{1}{x^3} - \dfrac{1}{x^2}$ have a global maximum over the interval $0 < x < \infty$? Explain your reasoning.

24. Consider a general quadratic function $f(x) = ax^2 + bx + c$ with coefficient $a \neq 0$.

(a) Find the critical point of the quadratic function $f(x) = ax^2 + bx + c$.

(b) Explain why any local extrema at the critical point is a global extrema over the interval $-\infty < x < \infty$.

25. A homeowner has 40 linear feet of fencing to enclose a rectangular garden.

(a) Find the largest area that can be enclosed. What are the corresponding dimensions of the garden?

(b) The homeowner wants the garden along a wall so that no fencing is required along the wall. Find the largest area that can be enclosed. What are the corresponding dimension of the garden?

26. Find two positive numbers x and y whose product in 784 and their sum is minimal.

27. A patient's temperature change T, in $^\circ F$, due to a dose d, in milligrams, of a drug is given by

$$T = 200d^2 - \frac{d^3}{3}$$

Find the dose which maximizes the temperature change.

28. Find the maximum volume of a closed box with a square base with a fixed surface area of $600\, in^2$.

29. An advertisement consists of a rectangular printed region plus 1 inch margins on the sides and 2 inch margins on the top and bottom. If the area of the printed region is to be 128 square inches, find the dimensions of the printed region that minimize the total area of the advertisement.

30. A can holds 12 fluid ounces (or 21.656 in^3). Find the dimensions that will minimize that amount of material used in the construction of the can.

[Hint. A cylinder of radius r and height h has a volume $V = \pi r^2 h$ and a surface area $S = 2\pi r^2 + 2\pi r h$.]

31. A patient has a circular skin infection due to an injury

(a) If the radius of the infected region is growing at a rate of 0.25 $\frac{mm}{day}$ when its radius is 3 mm, find the rate at which the area of the infected region is changing. Include units.

(b) If the area of the infected region is shrinking at a rate of 1 $\frac{mm^2}{day}$ when its radius is 4 mm, find the rate at which the radius of the infected region is changing. Include units.

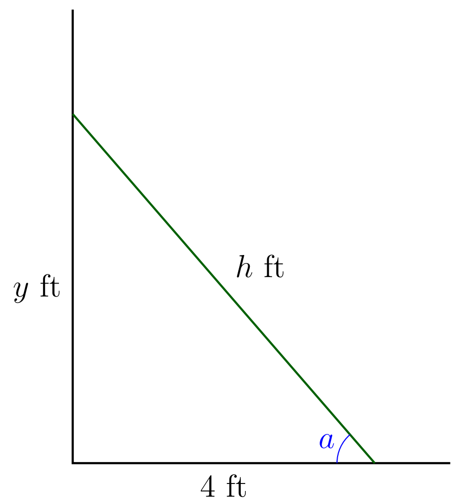
32. The Dubois formula relates a person's surface area, s in m^2, to their weight, w in kg, and height, h in cm, according to formula

$$s = 0.01w^{0.25}h^{0.75}.$$

(a) Assume a person maintains a constant weight of 50 kg and is 120 cm tall growing at a rate of 2 $\frac{cm}{year}$. At what rate is the surface area of the person changing? Include units.

(b) Assume a person is 120 cm tall growing at a rate of 2 $\frac{cm}{year}$ and weighs 50 kg losing weight at a rate of 3 $\frac{kg}{year}$. At what rate is the surface area of the person changing? Include units.

33. A ladder extended to length h $feet$ is leaning against a wall with its based fixed at 4 $feet$ from the wall and its top at height y along the wall. The angle a between the ground and the ladder is in *radians*.



(a) Find an equation that relates length h and height y.

(b) Find an equation that relates angle a and length h.

(c) If the ladder is extending at a rate of 0.2 $\frac{ft}{sec}$ when its length is 5 ft, find the rate at which height along the wall is changing. Include units.

(d) If the ladder is extending at a rate of 0.2 $\frac{ft}{sec}$ when its length is 8 ft, find the rate at which the angle is changing. Include units. [Hint. $a = \frac{\pi}{3}$ when $h = 8$. why?]

34. A vertical cylinder tank of radius 5 $meters$ and height 20 $meters$. If water is pumped into the tank at a rate of 3 $\frac{m^3}{min}$, find the rate at which the water level is rising when tank is half full.

35. A hemispherical bowl of radius 10 cm contains water to a depth of h cm.

(a) Find the radius r of the surface of the water as a function of the height h

(b) The water level drops at a rate of 0.1 $\frac{cm}{hour}$ when the height is 3 cm. Find the rate at which the radius of the surface of water is changing? Include units.