

As a reminder, Exam 2 cover Sections 2.1-2.6 and 3.1-3.4. A list of topics and suggested review problems is included below.

To access the review exercises, you'll need to log in to Wiley and go to the tab marked "Wiley Course Resources". Then, for the Chapter 2 exercises, click the down arrow next to "Ch 2: Key Concept: The Derivative (29)". The second item should be "Review Material and Projects: Chapter 02". Clicking this will bring up the review exercises as a downloadable pdf. For the Chapter 3 exercises, the process is similar.

The answers to all of the review questions will be posted in a separate pdf very soon.

Of course any other material you wish to study is also a good idea (old homework, quizzes, textbook, lecture notes, etc.).

Suggested review problems from the textbook

Ch 2
Rev Ex's # 3, 5, 7-10, 13-18, 20-24, 29, 33, 35, 36, 39, 41, 43, 45, 48, 50-55, 68

2.6 # 1, 2

Ch 3
Rev Ex's # 1-8, 10, 12, 15, 16, 24-26, 31, 41, 45-48, 51-54, 58-63, 67, 70, 73-75, 81, 82, 84-95,
96-97(a-e only)

Exam 2 Review Topics for MA 1450

Chapter 2: Limits and the Derivative

- Average velocity/average rate of change (§2.1-2.2)
- Instantaneous velocity/instantaneous rate of change (§2.1-2.2)
- Definition of the derivative of a function at a point (§2.2)
- Compute the derivative at a point by using the limit definition (§2.2)
- Relationship between the derivative and the slope of the tangent line (§2.2-2.3)
- Definition of the derivative of a function (§2.3)
- Compute the derivative of f by using the limit definition (§2.3)
- Given a graph of f , graph f' (§2.3)
- Relationship between the sign of f' and whether f is increasing/decreasing (§2.3)
- Interpret the meaning of the derivative in various contexts (§2.4)
- Definition of the second derivative (§2.5)
- Relationship between the sign of f'' and the concavity of f (§2.5)
- Continuity vs. smoothness (§2.6)

Chapter 3: Computing derivatives

- Derivatives of polynomials (§3.1)
- Derivatives of exponential functions (§3.2)
- Derivatives of products of functions (§3.3)
- Derivatives of quotients of functions (§3.3)
- Derivatives of compositions of functions (chain rule) (§3.4)

Individual Functions

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

$$\frac{d}{dx}[e^x] = e^x$$

$$\text{and } \frac{d}{dx}[a^x] = a^x \cdot \ln(a) \text{ for all } a > 0$$

Combining functions

$$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$\frac{d}{dx}[kf(x)] = kf'(x)$$

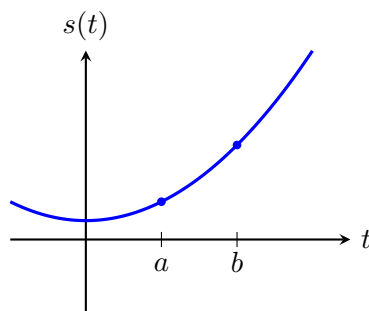
$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

Additional Review Problems

1. The graph of the position function $s = s(t)$ is below. Draw the graphical interpretation of average velocity over the time interval $a \leq t \leq b$. Draw the graphical interpretation for velocity at time $t = a$.

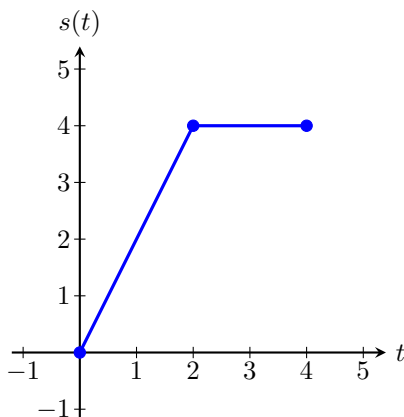


2. The table gives the position of a car, in miles, over the time interval $0 \leq t \leq 2$, in hours.

t (hours)	0	0.5	1	1.5	2
s (miles)	0	25	62	20	0

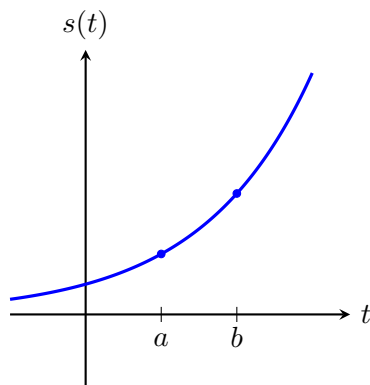
- (a) Find the average velocity of the car over the interval of time. Include units for the average velocity.
- (i) $0 \leq t \leq 0.5$, (ii) $0.5 \leq t \leq 1$, (iii) $1 \leq t \leq 2$.
- (b) Find the average velocity of the car over the interval $0 \leq t \leq 2$. What is this average velocity telling us about the car's journey?
3. At a time of t seconds, a particle moves a distance of $s(t) = 10t^2$ feet from its starting point.
- (a) Find the average velocity of the particle between time $t = 2$ and $t = 2 + h$ where:
- (i) $h = 0.01$ (ii) $h = 0.001$ (iii) $h = 0.0001$
- (b) Use Part (a) to conjecture the velocity of the particle at $t = 2$ seconds. Include units.
- (c) Use the limit definition of velocity to find the velocity of the particle at $t = 2$ seconds. Include units.

4. Below is the graph of the position $s = s(t)$, in miles, of an object at t hours from its start point.

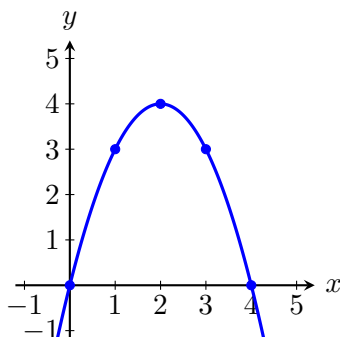


Without direct computations, explain why the velocity at time $t = 1$ hour is greater than the average velocity over the interval $0 \leq t \leq 4$.

5. The graph of the function $y = f(x)$ is below. Draw the graphical interpretation of the average rate of change over the time interval $a \leq x \leq b$. Draw the graphical interpretation for the derivative at a .

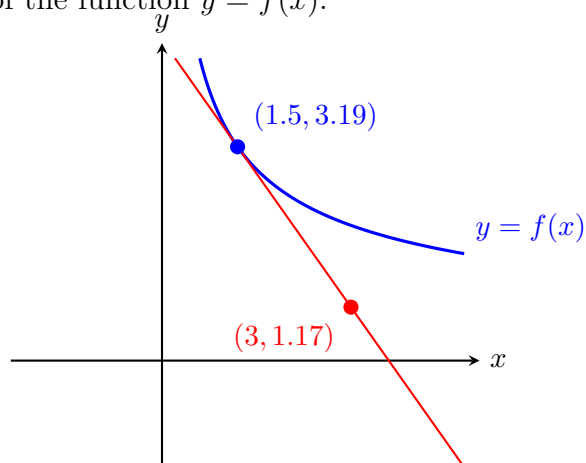


6. Below is the graph of the function $y = f(x)$.

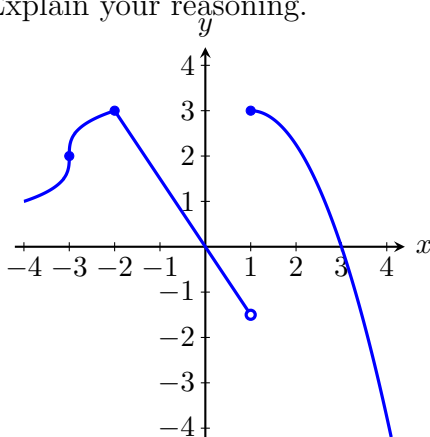


- (a) Find the average rate of change over the interval $1 \leq x \leq 4$.
- (b) Write the following quantities in ascending order: $f'(0)$, $f'(2)$, $f'(4)$
 $\frac{f(2) - f(1)}{2 - 1}$, $\frac{f(4) - f(2)}{4 - 2}$.

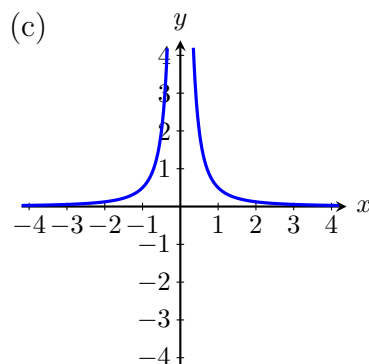
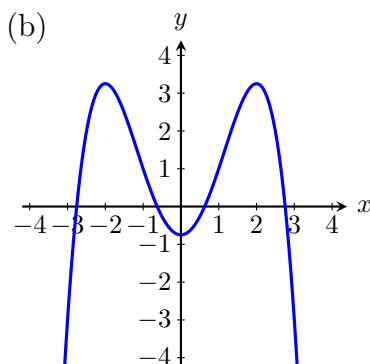
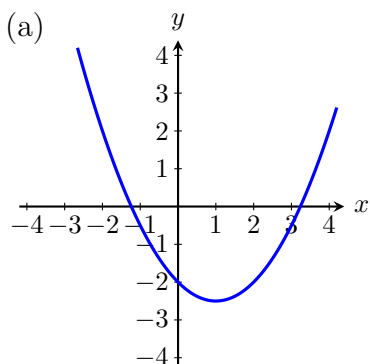
7. Below is the graph of the function $y = f(x)$.



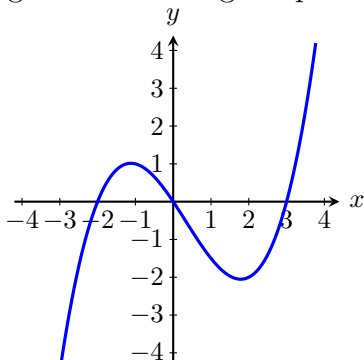
- (a) Find $f'(1.5)$. Write an equation of the tangent line to the function $y = f(x)$ at $a = 1.5$.
- (b) Use the tangent line in Part (a) to estimate $f(1.75)$.
8. Use the limit definition of derivative to compute $f'(a)$ when:
- (a) $f(x) = 3x - x^2$ with $a = -1$ (c) $f(x) = \sqrt{x+1}$ with $a = 3$
- (b) $f(x) = \frac{4}{x}$ with $a = 2$ (d) $f(x) = \frac{1}{x^2}$ with $a = 1$
9. Below is the graph of the function $y = f(x)$. Find all x where the function $y = f(x)$ fails to be differentiable. Explain your reasoning.



10. Given the graph of the function, sketch a graph of its derivative.



11. Below is the graph of the derivative $\frac{dy}{dx} = f'(x)$. Determine the intervals where the function $y = f(x)$ is increasing and decreasing. Explain your reasoning.



12. Compute the derivative of the function using the limit definition.

(a) $f(x) = 3x - x^2$

(c) $f(x) = \sqrt{x+1}$

(b) $f(x) = \frac{4}{x}$

(d) $f(x) = \frac{1}{x^2}$

13. A company's costs, in dollars, to make r t-shirts is $C = f(r)$.

(a) What are the units of $\frac{dC}{dr}$?

(b) Explain the meaning of $f(500) = 125$ and $f'(500) = 0.30$. Include units in your explanation.

(c) Use Part (a) to approximate the cost for the company to make 550 t-shirts.

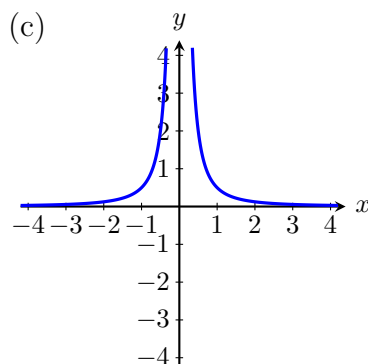
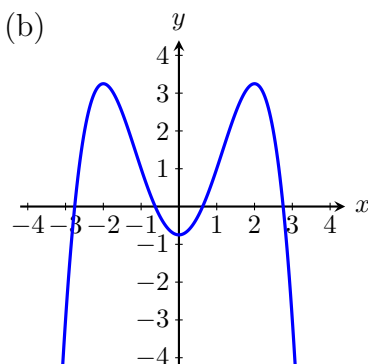
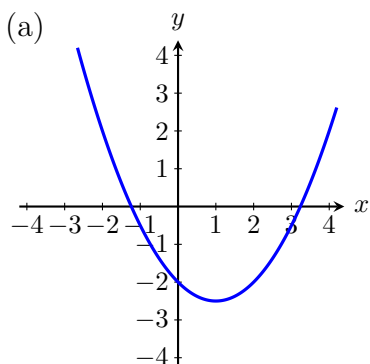
14. The temperature T , in Celsius, of the air at the altitude z kilometers above the ground is given by $T = g(z)$.

(a) What are the units of $\frac{dT}{dz}$?

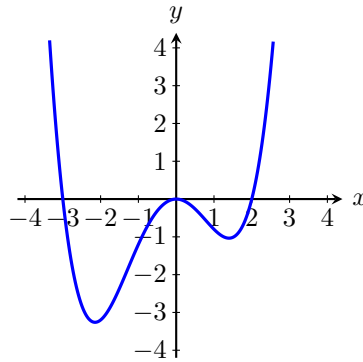
(b) Explain the meaning of $g(2) = 25$ and $g'(2) = -6.5$. Include units in your explanation.

(c) Explain the meaning of the function $z = g^{-1}(T)$. What are the units of $\frac{dz}{dT} = (g^{-1})'(T)$?

15. Sketch the second derivative of each function graphed below.



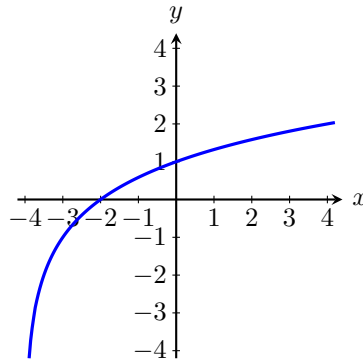
16. Below is the graph of the function $y = f(x)$. Assume that $f''(-1) \neq 0$.



Determine whether quantity is positive, negative or zero. Explain your reasoning.

- (a) $f(-1)$ (b) $f'(-1)$ (c) $f''(-1)$

17. Below is the graph of the derivative $y' = f'(x)$.



- (a) Is the function $y = f(x)$ increasing or decreasing over the interval $0 < x < 2$? Explain your reasoning.
- (b) Is the function $y = f(x)$ concave up or concave down over the interval $0 < x < 2$? Explain your reasoning.
18. Sketch the graph of a function $y = f(x)$ which satisfies the following conditions:
- (a) $f'(x) < 0$ and $f''(x) > 0$ over the interval $-\infty < x < \infty$.
- (b) $f'(x) < 0$ and $f''(x) < 0$ over the interval $-\infty < x < \infty$.
19. The position s , in feet, of a particle at time t , in seconds, is given by $s(t) = 12t^3 + t$.
- (a) Find the velocity $v = v(t)$ and the acceleration $a = a(t)$ at any time t . [You may use the differentiation rules established in Chapter 3.]
- (b) Interpret $s(2)$, $v(2)$ and $a(2)$. Include units.
20. Find the derivative of each function.
- (a) $y = x^{7.8}$ (c) $Q = \sqrt[7]{x^3}$
- (b) $g(t) = \frac{1}{t^5}$ (d) $f(x) = \frac{1}{\sqrt[7]{x}}$

21. Find the derivative of each function.

(a) $f(x) = 12x^5 - 3x^2 + 4$

(e) $y = 4 \cdot 6^x - 5x^3 + 6$

(b) $P = \frac{2}{z^{6.5}} - 12$

(f) $f(t) = \sqrt{t} - 10e^t$

(c) $y = \sqrt[3]{t} - \frac{1}{\sqrt[3]{t}} - \frac{3}{t^3}$

(g) $y = e^x + x^e + e^2$

(d) $f(x) = \frac{3x - 4}{x^2}$

(h) $P = \frac{8}{r^4} - \frac{4^r}{8} + 8$

22. Let $f(x) = 1 - \sqrt{x}$.

(a) Sketch a graph of the function $f(x) = 1 - \sqrt{x}$ and its tangent line at $a = 4$.

(b) Find an equation of the tangent line of $f(x) = 1 - \sqrt{x}$ at $a = 4$.

23. Let $f(x) = 1 - 2e^x$.

(a) Find the x where the tangent line to $f(x) = 1 - 2e^x$ has slope -2 .

(b) Find an equation of the tangent line described in Part (a).

24. Find the second derivative of the function $f(x) = 2 \cdot 10^x$. Use the second derivative to determine whether the function $f(x) = 3^x$ is concave up or concave down on the interval $-\infty < x < \infty$.

25. The height of a sand dune, in centimeters, at time t years from 2005, is given by $h(t) = 700 - 3t^2$. Find $f(5)$ and $f'(5)$. Explain the meaning of $f(5)$ and $f'(5)$. Include units in the explanation.

26. The value of a car, in thousands of dollars, purchased in 2020 is modeled $V(t) = 35(0.85)^t$, where t is years from date of purchase.

(a) Find $\frac{dV}{dt}$.

(b) Find $V(5)$ and $V'(5)$. Explain the meaning of $V(5)$ and $V'(5)$. Include units in the explanation.

27. Assume that $f'(2) = 5$ and $g'(2) = -3$

(a) Find $H'(2)$ when $H(x) = 1 + 4f(x) - g(x)$.

(b) Find $G'(2)$ when $G(x) = 2^x - 10g(x)$.

28. Find the derivative of each function.

(a) $f(x) = x^2 3^x$

(b) $y = (3x^6 - 7x + 2)(x^{10} + 8x^5)$

(c) $y = \frac{9 - \sqrt[3]{x}}{x^3}$

(d) $g(t) = \frac{t^2 - 1}{4e^t + 1}$

(e) $P = r^8 - \frac{8^r}{r + 8^r}$

(f) $y = 2(4x^2 - 3x + 4)^{10}$

(g) $f(x) = \sqrt{1 + 2x^8}$

(h) $g(t) = \frac{t^2}{4} - e^{\sqrt{t}}$

(i) $P = \frac{10z^5}{e^{z^2}}$

(j) $v = \frac{e^{-t}}{1 - e^{-t}}$

29. Find an equation of the tangent line to the function $f(x) = 2xe^x$ at $a = 0$.

30. Find an equation of the tangent line to $g(x) = (4x - 1)^3$ at $a = \frac{1}{2}$.

31. Find the second derivative of $f(x) = \frac{4x - 1}{x}$. Is the function concave up or concave down on the interval $(0, \infty)$? Explain your reasoning. [Hint. Simplify $f'(x)$ before computing $f''(x)$.]

32. Find the second derivative of the function $f(x) = e^{-x^2}$.

33. The world's population, in billions, is modeled by $P = 6e^{0.013t}$, where t is years since 1999.

(a) Find $\frac{dP}{dt}$.

(b) Find $P(10)$ and $P'(10)$. Explain the meaning of $P(10)$ and $P'(10)$. Include units in the explanation.

34. Use the following chart to evaluate the indicated derivative.

x	1	6
$f(x)$	6	5
$f'(x)$	-1	8
$g(x)$	7	3
$g'(x)$	-5	-2

(a) $H'(1)$ when $H(x) = f(x)g(x)$

(b) $P'(1)$ when $P(x) = \frac{f(x)}{g(x)}$

(c) $h'(6)$ when $h(x) = 3x^2 f(x)$

(d) $k'(6)$ when $k(x) = \frac{4x^2}{g(x)}$

(e) $G'(6)$ when $G(x) = (1 + g(x))(1 - g(x))$

(f) $q(1)$ when $q(x) = g(f(x))$

(g) $h'(1)$ when $h(x) = \sqrt{f(x)}$

(h) $H'(1)$ when $H(x) = f(\sqrt{x})$

(i) $F'(6)$ when $F(x) = e^{5g(x)}$

(j) $Q'(6)$ when $Q(x) = e^{-x} g(x)$