

As a reminder, Exam 1 will cover Sections 1.1-1.9. A list of topics and review problems is included below.

To access the review exercises, you'll need to log in to Wiley and go to the tab marked "Wiley Course Resources". Then, for the Chapter 1 exercises, click the down arrow next to "Ch 1: Foundation for Calculus: Functions and Limits (45)". The second item should be "Review Material and Projects: Chapter 01". Clicking this will bring up the review exercises as a downloadable pdf. Alternatively, they can be found at the end of the chapter in the ebook available online.

The answers to all of the odd-numbered questions from the individual sections are available in the textbook. The answers to the even-numbered questions will be included in a separate pdf and will be available soon.

Of course any other material you wish to study is also a good idea (old homework, quizzes, textbook, lecture notes, etc.).

Exam 1 Review Topics for MA 1450

- Equations of lines (point-slope and slope-intercept) (§1.1)
- Definition of a function (§1.1)
- Domain and range of a function (§1.1)
- Composition of functions (§1.3)
- Definitions of increasing and decreasing (§1.1)
- Exponential functions (§1.2)
- Logarithmic functions (§1.4)
- Exponential and logarithmic rules (§1.4)
- How to solve an exponential equation (§1.4)
- Definition of radians (§1.5)
- Definition of sine/cosine (§1.5)
- End behavior of polynomial and rational functions (§1.6)
- Horizontal/vertical asymptotes (§1.6)
- Dominance of functions (§1.6)
- Idea of continuity of a function on an interval (§1.7)
- Definition of a limit and how to compute them (§1.7-1.9)

- Definition of continuity (§1.7)
- Compute a limit from a graph (§1.7-1.8)
- Properties of limits (§1.8)
- Properties of continuous functions (§1.8)
- One-sided limits (§1.8)
- Use algebra to simplify and compute limits (§1.9)
- The squeeze theorem (§1.9)

Suggested review problems from the textbook

Ch 1
Rev Ex's # 1, 3, 6, 7, 8, 10-13, 15-17, 25-32, 34, 36-38, 42-44, 47-52, 64, 67-72, 79, 85, 87, 92, 99,
 101, 102, 107, 115, 117, 119

1.1 # 17, 18

1.3 # 1, 8, 41, 42

1.4 # 1-6, 30, 31

1.5 # 1, 3, 5, 7, 9

1.7 # 1, 5

1.8 # 1, 3, 67

Note: The absence of exercises from sections 1.2, 1.6, and 1.9 is due to the fact that all the necessary material covered in those sections is included in the Chapter 1 Review Exercises. You are still responsible for the material from all of the sections. Said differently, the only reason 1.1, 1.3, 1.4, 1.5, 1.7, and 1.8 have additional exercises is because there are not enough review exercises in the book to adequately cover the topics from those sections.

Additional Review Problems

1. Find the domain of each function below.

(a) $f(x) = \frac{1}{x^2 - 4}$

(b) $g(t) = \sqrt{t - 1}$

2. The value of a house $V = f(t)$, in thousands of dollars, is a function of time t , in years, since its purchase.

(a) Interpret $f(0) = 200$ and $f(10) = 325$. Include units in the interpretation.

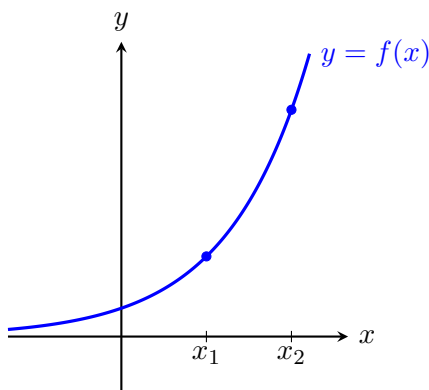
(b) Sketch a possible graph of the function $V = f(t)$. Interpret what your graph is saying about the value of the house over time.

3. Consider a box with width $w = 4$ in, length $\ell = 6$ in and height h in.

(a) Write the volume of the box as a function of its height h .

(b) Find a reasonable domain for the function found in Part (a).

4. Given the graph of the function $y = f(x)$ below, draw the visual interpretation of the difference quotient $\frac{\Delta y}{\Delta x}$ using inputs x_1 and x_2 .



5. Consider the line with slope m which passes through the point (x_0, y_0) .

(a) Find a formula for the line.

(b) Find the y -intercept of the line.

6. Consider the line which passes through the points $(-1, 6)$ and $(5, -2)$.

(a) Sketch a graph of the line.

(b) Find an equation of the line.

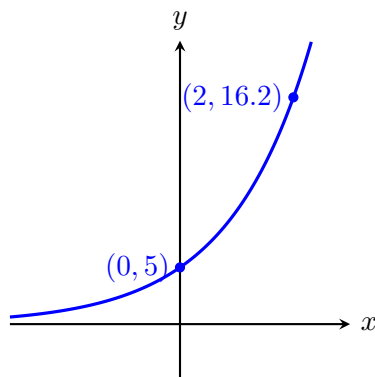
(c) Find the y -intercept of the line.

7. A linear function generates the following table:

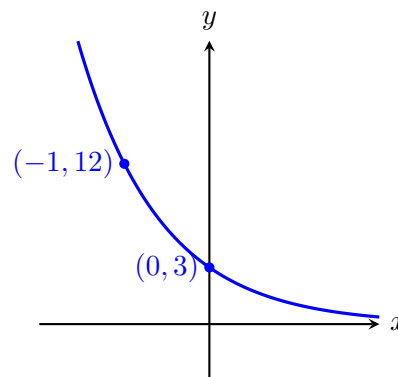
x	5.2	5.4	5.6	5.8
$f(x)$	12.1	12.5	12.9	13.3

- (a) Find an equation of the linear function.
 (b) What is the y -intercept of the line?
8. Each of the following graphs represent an exponential function. Find a formula for each function.

(a)



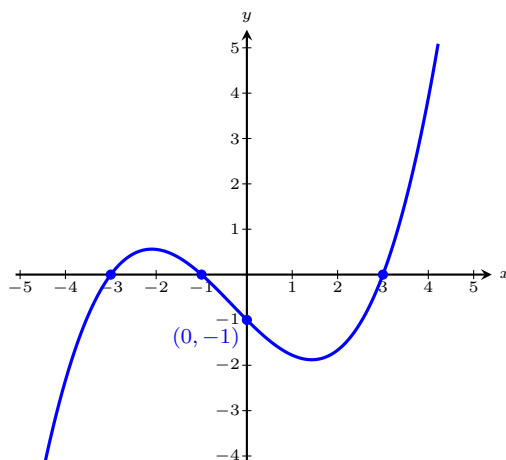
(b)



9. In 2007, the world's population reached 6.7 billion and was increasing at a rate of 1.2% per year. Write the population $P(t)$ as a function of time t in years since 2007. Then predict the world's population in 2027.
10. Find a formula for the exponential function whose graph passes through the points $(1, 16)$ and $(-2, 2)$.
11. Consider the functions $z = g(x)$ and $y = f(z)$.
- (a) Which variables represent the inputs of the functions $z = g(x)$ and $y = f(z)$. What about the outputs?
- (b) Explain how outputs of the function composition $y = f(g(x))$ are related to the inputs of the functions $z = g(x)$ and $y = f(z)$.
12. Let $f(x) = x^2 + 4$ and $g(x) = 1 + \sqrt{x}$. Compute the following:
- (a) $f(g(0))$
 (b) $f(g(x))$
 (c) $g(f(x))$
 (d) $g(a + h) - g(a)$
13. Repeat Problem 12 with $f(x) = \frac{1}{x}$ and $g(x) = 5x - 1$.

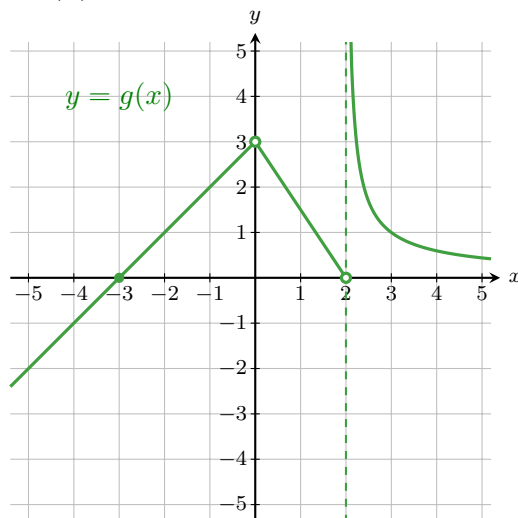
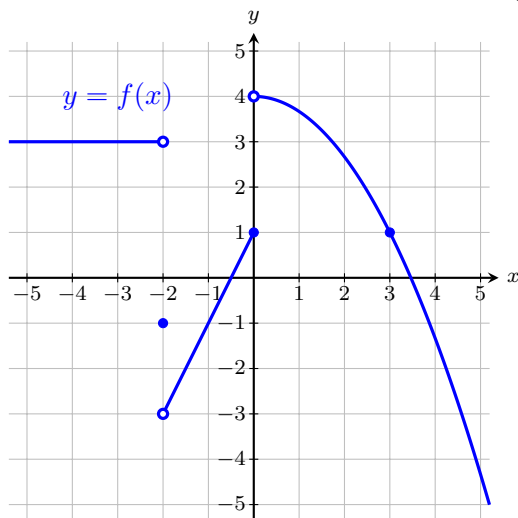
14. Simplify the expression $\frac{f(x+h) - f(x)}{h}$ for the given function.
- (a) $f(x) = 5x - x^2$
 - (b) $f(x) = \frac{1}{x}$
15. Use logarithms to solve for x .
- (a) $10^{5x} = 8$
 - (b) $6 + 4 \cdot 3^{x-1} = 8$
 - (c) $12 \cdot 10^x = 4 \cdot 2^x$
16. The initial population of a bacteria colony is 40 thousand bacteria. In 5 hours, bacteria colony population decreases to 35 thousand bacteria. Assume the population of the bacteria colony is decreasing exponentially. Find when the population will be half its initial size.
17. Explain visually the definition of $\sin(\theta)$ and $\cos(\theta)$ using the unit circle.
18. Sketch the graphs of $y = \sin(x)$, $y = \cos(x)$.
19. Draw the angle in a unit circle. Without using a calculator, determine whether sine and cosine are positive, negative, zero or undefined.
- (a) $\pi/3$
 - (b) $-\pi/2$
 - (c) 4
20. Determine the end behavior of each polynomial shown below.
- (a) $p(x) = 7x^5 + 1$
 - (b) $q(x) = 1 + 2x - 3x^4$

21. The graph of a polynomial function p is shown below. Find a formula for $p(x)$.



22. Explain the connection between the two-sided limit $\lim_{x \rightarrow c} f(x)$ and the one-sided limits $\lim_{x \rightarrow c^+} f(x)$ and $\lim_{x \rightarrow c^-} f(x)$.

23. Use the graph of the functions $y = f(x)$ and $y = g(x)$ to determine the following limits:



(a) $\lim_{x \rightarrow 3} f(x)$

(f) $\lim_{x \rightarrow -2} f(x)$

(k) $\lim_{x \rightarrow \infty} f(x)$

(b) $\lim_{x \rightarrow 3^+} f(x)$

(g) $\lim_{x \rightarrow 2^+} g(x)$

(l) $\lim_{x \rightarrow -\infty} f(x)$

(c) $\lim_{x \rightarrow 3^-} f(x)$

(h) $\lim_{x \rightarrow 2^-} g(x)$

(m) $\lim_{x \rightarrow 0^+} [5f(x) + g(x)]$

(d) $\lim_{x \rightarrow -2^+} f(x)$

(i) $\lim_{x \rightarrow 2} g(x)$

(n) $\lim_{x \rightarrow 0} \frac{g(x)}{x+1}$

(e) $\lim_{x \rightarrow -2^-} f(x)$

(j) $\lim_{x \rightarrow \infty} g(x)$

(o) $\lim_{x \rightarrow -1} 3f(x)g(x)$

24. Use numerical evidence to conjecture the value of the limit $\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x}$. Use a table which includes $x = 0.1, 0.01, 0.001, 0.0001$ and $x = -0.1, -0.01, -0.001, -0.0001$.

25. Find each limit by looking at the graph of a function.

(a) $\lim_{x \rightarrow 0} \frac{1}{x^2}$ and $\lim_{x \rightarrow \infty} \frac{1}{x^2}$

(b) $\lim_{x \rightarrow 1} \frac{|x - 1|}{x - 1}$ and $\lim_{x \rightarrow \infty} \frac{|x - 1|}{x - 1}$

26. Use properties of limits to find the limit algebraically. Show all work to justify answer (as demonstrated during lectures).

(a) $\lim_{x \rightarrow -1} [3x^4 - x + 2]$

(b) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x + 2}$

(c) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

(d) $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 9}$

(e) $\lim_{x \rightarrow 16} \frac{\sqrt{x} - 4}{x - 16}$

(f) $\lim_{h \rightarrow 0} \frac{(5 + h)^2 - 25}{h}$

27. Use properties of limits to compute the limit algebraically. Show all work to justify answer (as demonstrated during lectures).

(a) $\lim_{x \rightarrow \infty} \frac{10x^5 - 4x + 1}{1 - 2x^5}$

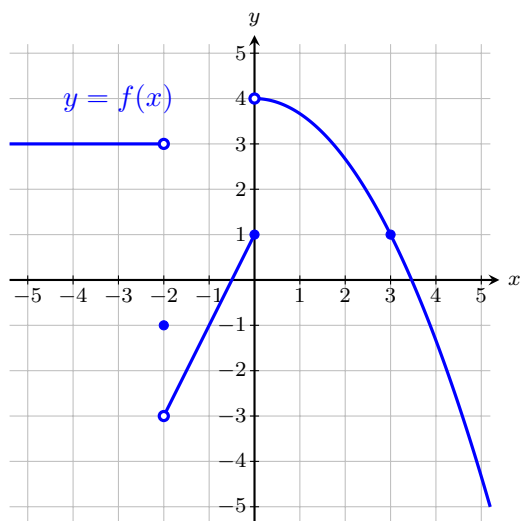
(b) $\lim_{x \rightarrow -\infty} \frac{2x^4 - 4x + 1}{x^6 + 1}$

(c) $\lim_{x \rightarrow \infty} \frac{e^x}{1 + 2e^x}$

(d) $\lim_{x \rightarrow -\infty} \frac{e^x}{1 + 2e^x}$

28. (a) State the limit definition of a function $f(x)$ being continuous at $x = c$.
 (b) Explain the definition of f being continuous at $x = c$ using the graph of $y = f(x)$.
 (c) State the definition of the function $y = f(x)$ being continuous on an interval.
 (d) Explain the definition of continuous on an interval using the graph of the function $y = f(x)$.
 (e) List which “classes” of functions are continuous on the interval $(-\infty, \infty)$.

29. Consider the function $y = f(x)$ whose graph is shown below.



- (a) Find all c for which the function $y = f(x)$ is discontinuous at $x = c$. Explain your reasoning.
- (b) Is the function continuous on the interval $0 < x < \infty$? Explain your reasoning.
30. Let $y = f(x)$ be a function such that $4x \leq f(x) \leq x^2 + 2x + 1$ for all x . Use the Squeeze Theorem to find $\lim_{x \rightarrow 1} f(x)$.
31. Let $f(x) = \begin{cases} kx^2, & \text{if } x < 2 \\ 3, & \text{if } x \geq 2 \end{cases}$. Find the constant k for which the function $y = f(x)$ is continuous at $c = 2$.
32. (a) Find examples of two functions $y = f(x)$ and $y = g(x)$ such that $\lim_{x \rightarrow 0} f(x) = \infty$ and $\lim_{x \rightarrow 0} g(x) = \infty$ but $\lim_{x \rightarrow 0} [f(x) - g(x)] = 2$.
- (b) Find examples of two functions $y = f(x)$ and $y = g(x)$ such that $\lim_{x \rightarrow \infty} f(x) = \infty$ and $\lim_{x \rightarrow \infty} g(x) = \infty$ but $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 10$.