MATH 2100 – HOMEWORK 6

Fall 2022

due Wednesday, **December 7**, at the start of class

Sections 3.3, 4.1, 4.2, 4.3, 4.4

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. Please write the questions in the correct order. Explain all reasoning.

1. Prove that $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$.
2. Prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$.
3. Draw the two-sided arrow diagram for the function $f : \mathcal{P}(\{4,5,6\}) \to \mathcal{P}(\{1,2,3\})$ defined by
$$f(S) = \{x - 3 : x \in S\} \setminus \{2\}.$$
4. Prove that the function $h : \mathbb{N} \to \mathbb{N}$ defined by $h(n) =$ [the sum of the digits in n (in base 10)] is surjective. Prove that it's not injective.
5. Let $c : \mathcal{P}(\{x,y,z\}) \to \mathcal{P}(\{x,y,z\})$ be the function with the rule $c(A) = \{x,y,z\} \setminus A$, and let $n : \mathcal{P}(\{x,y,z\}) \to \{0,1,2,3\}$ be the function such that $n(A)$ is the number of elements in the set A. Which composition makes sense, $c \circ n$ or $n \circ c$? For the one that is defined, give the domain, codomain, range, and draw the two-sided arrow diagram.
6. Let $A = \{0,1,2,3\}$ and let $B = \{000,001,010,011,100,101,110,111\}$ be the set of binary strings with three digits. Define $g : B \to A$ by $g(s) =$ [the number of 1s in s]. Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).
7. ~~Consider the set $S = \mathcal{P}(\{1,2,3,4\})$. Define $\Sigma(T)$ to be the sum of the elements in T. For example $\Sigma(\{1,3,4\}) = 8$. Define the relation $R = \{(A,B) \in S \times S : \Sigma(A) < \Sigma(B)\}$. Answer the following questions.~~
   (a) ~~Is R reflexive?~~
   (b) ~~Is R irreflexive?~~
   (c) ~~Is R symmetric?~~
   (d) ~~Is R antisymmetric?~~
   (e) ~~Is R transitive?~~
   (f) ~~Is R a partial order? If so, draw the Hasse diagram.~~
8. ~~Let $S = \{0,1,2,3,4,5,6,7,8,9,10\}$ and $R = \{(a,b) \in S \times S : |a - b| = 4\}$. Answer questions (a) – (f) from Question 6.~~
9. ~~Let $S = [0,4\pi)$ and define the relation $R = \{(a,b) \in S \times S : \sin(a) = \sin(b)\}$. Answer questions (a) – (f) from Question 6.~~

10. ~~Let $A = \{1, 4, 7\}$. Give an example of a relation R on A that is~~

   (a) ~~Transitive and reflexive but not antisymmetric.~~

   (b) ~~Antisymmetric and reflexive but not transitive.~~

   (c) ~~Antisymmetric and transitive but not reflexive.~~