MATH 2100 – EXAM 3

Friday, December 2

Name: _____

Instructions: Please write your work neatly and clearly. **You must explain all reasoning. It is not sufficient to just write the correct answer.** You have 50 minutes to complete this exam. You may not use calculators, notes, or any other external resources.

Scores

1	
2	
3	
4	
5	
6	

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- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

If you understand and agree to abide by this honor code, sign here:

_____

1. Use Venn Diagrams to check whether or not the property below is true. Additionally, if it's false, give a specific counterexample that demonstrates that it's false.

For all sets A, B, and C: $(B \cap C) - A = B \cap (C - A)$.

2. Let $A = \{1, 2, 3, 4, 5\}$. List five elements of each of the following sets.

   (a) $\{x \in \mathcal{P}(A) : |x| = 2\}$

   (b) $\{(x, y) \in A \times A : x \leq y\}$

   (c) $\{(x, y) \in \mathcal{P}(A) \times \mathcal{P}(A) : x \subseteq y\}$

3. Prove that if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

4. Prove the following identity:

$$(\{2n+1 : n \in \mathbb{Z}\} \cap \{5m+4 : m \in \mathbb{Z}\}) \subseteq \{10k+9 : k \in \mathbb{Z}\}$$

5. Consider the function $q : \mathcal{P}(\{2,3,5\}) \to \mathbb{N}$ defined by

$$q(S) = [\text{the product of the elements of } S]$$

and $q(\emptyset) = 1$. For example $q(\{2,5\}) = 2 \cdot 5 = 10$.

(a) What is the domain of q?

(b) What is the codomain of q?

(c) What is the range of q?

(d) Draw the two-sided arrow diagram of q. (You obviously don't need to draw the whole codomain, but make sure at least the whole range is included.)

(e) Is q invertible? (Be sure to justify your answer.)

6. Define functions $f : \mathbb{Z} \to \mathbb{R}$ and $g : \mathbb{N} \to \mathbb{Z}$ by

$$f(n) = \sqrt{|n|} \qquad \text{and} \qquad g(m) = 5 - m^2.$$

(a) Does $f \circ g$ make sense? If so, compute it and state its domain and codomain. If not, explain why not.

(b) Does $g \circ f$ make sense? If so, compute it and state its domain and codomain. If not, explain why not.