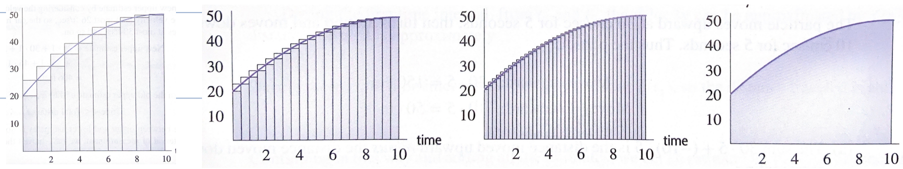


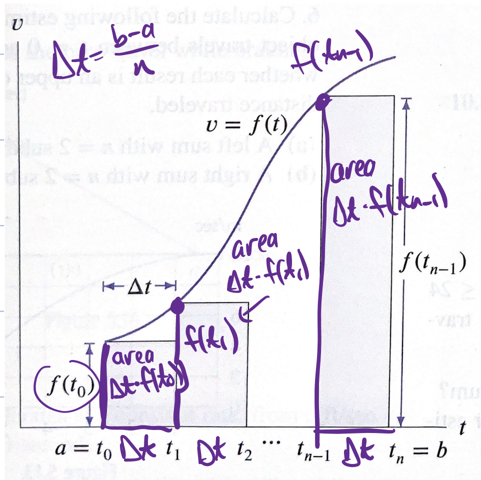


Figure 5.8: Left-hand sums

Wednesday, Dec 7 – Fall '22
Lecture #40

Announcements / Reminders

* Wiley Plus #14 due tonight
* Quiz 12 in discussion tomorrow

} (4.7, 5.1)

until Sunday

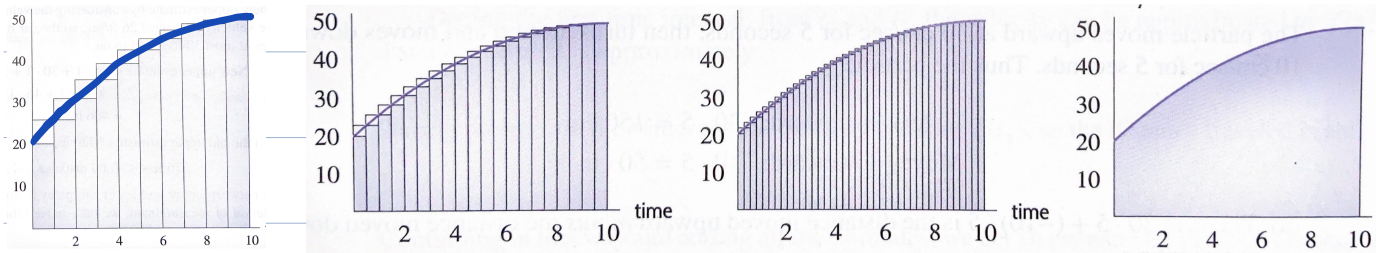
* Course Evaluations are open. If 90% of the class does them, I will give everyone two bonus points.

* Final Exam, 1pm–3pm on Monday, in this room

Section 5.2 – The Definite Integral

In 5.1: change in position
= (signed) area under the velocity curve

and this can be approximated by adding the areas of a bunch of rectangles



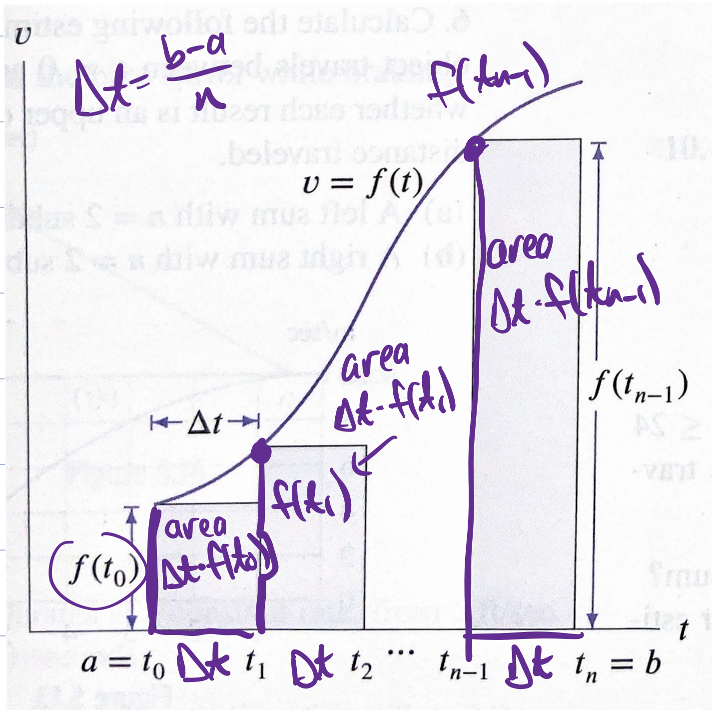


Figure 5.8: Left-hand sums

From a to b with n rectangles, the width of each is:

$$\Delta t = \frac{b-a}{n}$$

Left sum: $\Delta t \cdot \left(f(t_0) + f(t_1) + \cdots + f(t_{n-1})\right)$

Right sum: $\Delta t \cdot \left(f(t_1) + f(t_2) + \cdots + f(t_n)\right)$

To write summations more concisely, we use something called "Sigma notation".

"Σ" is a capital greek sigma. σ

Let "$P(i)$" be some mathematical expression involving the variable i.

upper bound → $i=10$

$$\sum_{i=1}^{i=10} P(i) \quad \text{means}$$

lower bound → $i=1$

"$P(1)+P(2)+P(3)\cdots\cdots+P(10)$"

Example:

$$\sum_{i=3}^{6} i^2 = 3^2+4^2+5^2+6^2 = 86$$

$$\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\cdots\cdots+n^2$$

$\boxed{\text{LH sum:}}$ $\Delta t\cdot(f(t_0)+f(t_1)+\cdots f(t_{n-1}))$

$= \Delta t\cdot f(t_0) + \Delta t\cdot f(t_1) + \cdots\cdots$

$+ \Delta t\, f(t_{n-1})$

$$= \boxed{\sum_{i=0}^{n-1} (\Delta t\cdot f(t_i))} = \Delta t\cdot f(t_0) + \Delta t\cdot f(t_1) + \cdots\cdots$$

$+\Delta t \cdot f(t_{n-1})$.

RH sum:

$$\sum_{i=\cancel{0}\,1}^{\cancel{n-1}\,n} \left(\Delta t \cdot f(t_i)\right)$$

n is the # of rectangles used in the approximation. The larger n gets, the better the approximation is.

So, we'll take the limit as $n \to \infty$ to get the exact area under the curve instead of an approximation.

The Definite Integral

We use the notation "$\int_a^b f(t)\,dt$" to

mean: the (signed) area under the curve $f(t)$ between $t=a$ and $t=b$.

$\frac{dy}{dx}$ } not a real fraction

right endpoint → b

the function we care about

$$\int_a^b f(t)\,dt$$

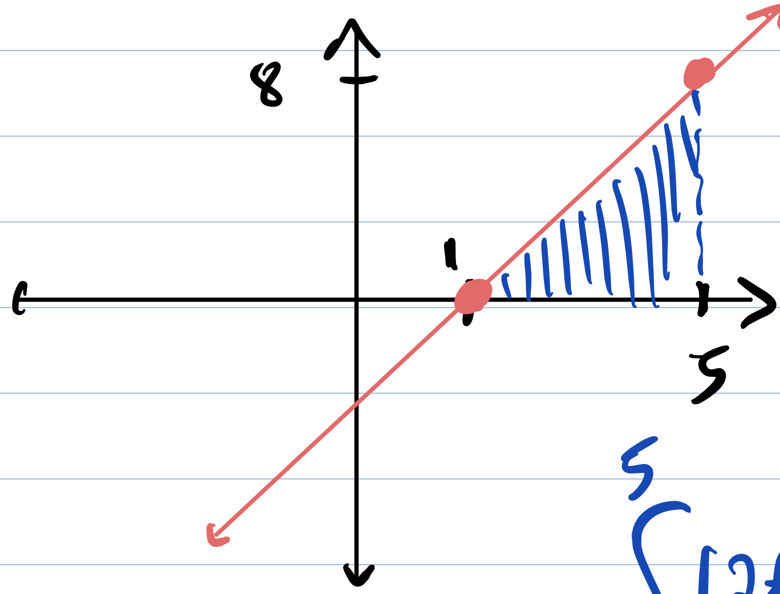
left endpoint → a

a piece of the notation telling us which variable is the main one

not a real thing you're multiplying by

Ex:

$$\int_1^5 (2t-2)\,dt = \text{(signed) area under } 2t-2 \text{ from } t=1 \text{ to } t=5$$



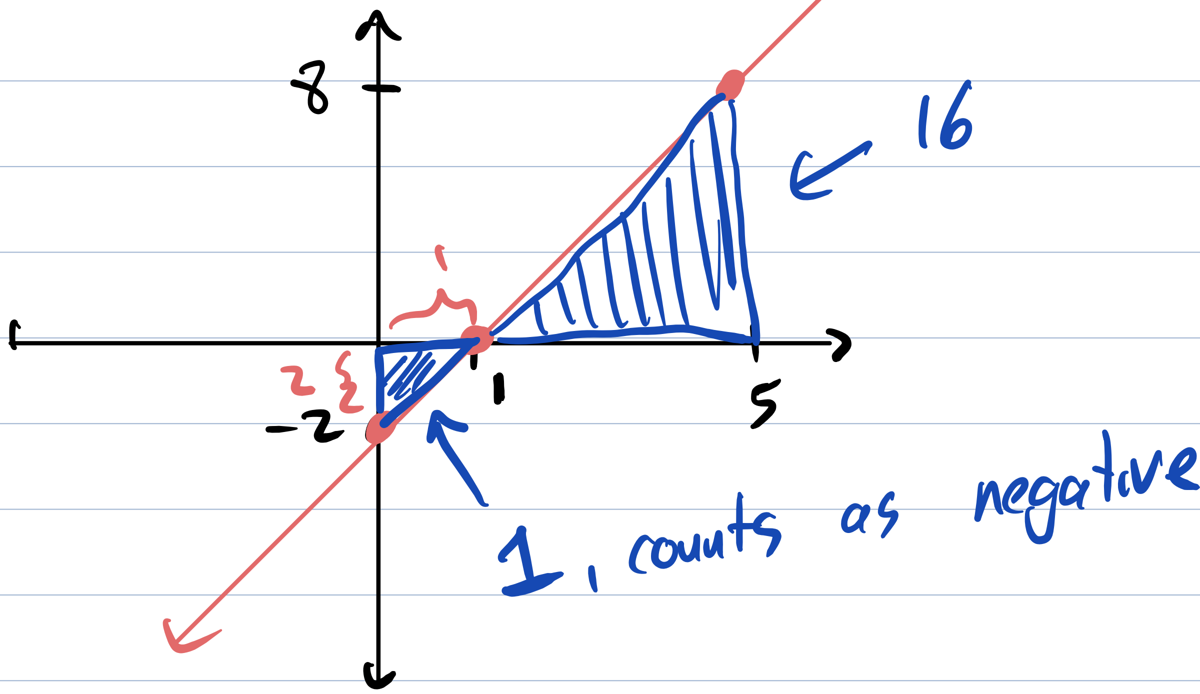
base $= 5-1=4$

height $=8$

area $= \frac{1}{2}\cdot b\cdot h = 16$

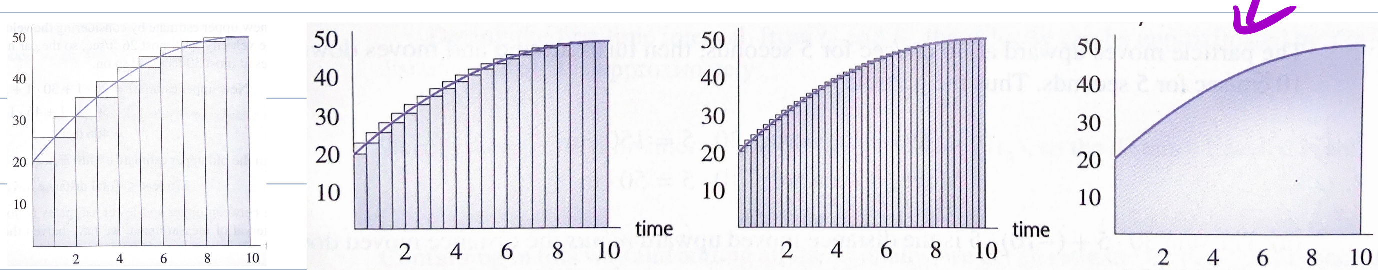
$$\int_1^5 (2t-2)\,dt = 16$$

Ex: $\displaystyle\int_0^5 (2t-2)\,dt = 16 - 1 = 15$

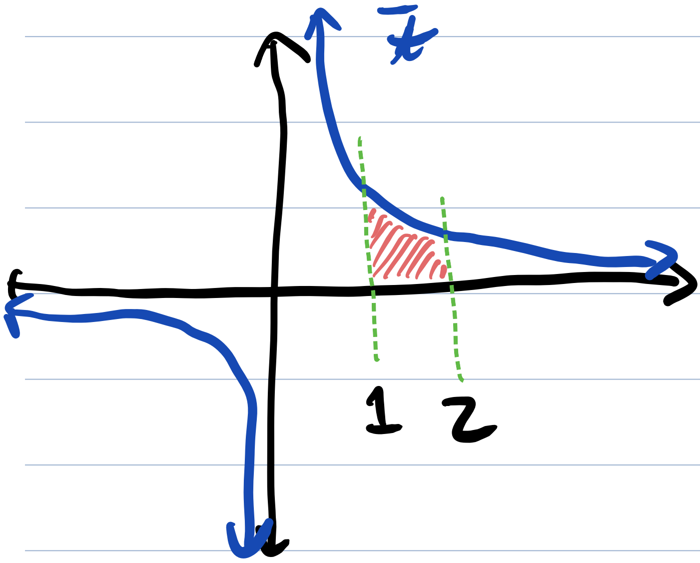


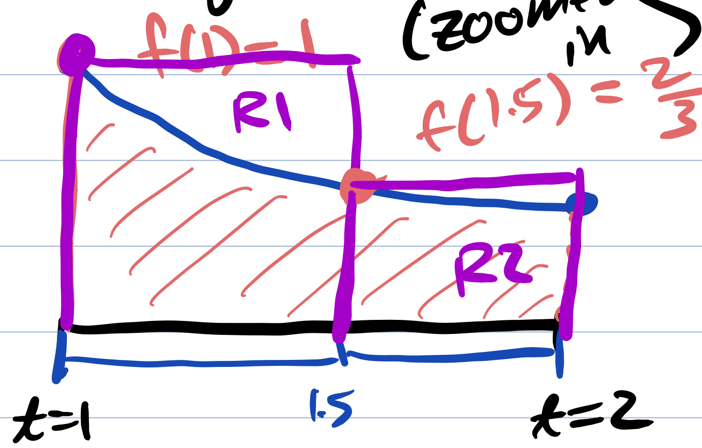
Limit Definition of an integral:

$$\int_a^b f(t)\,dt = \lim_{n\to\infty}\left(\sum_{i=0}^{n-1}\left(\Delta t\cdot f(t_i)\right)\right)$$



Ex: Estimate $\int_1^2 \frac{1}{t}\,dt$ with a LH sum with $n=2$ rectangles





$\Delta t = \frac{2-1}{2} = \frac{1}{2}$

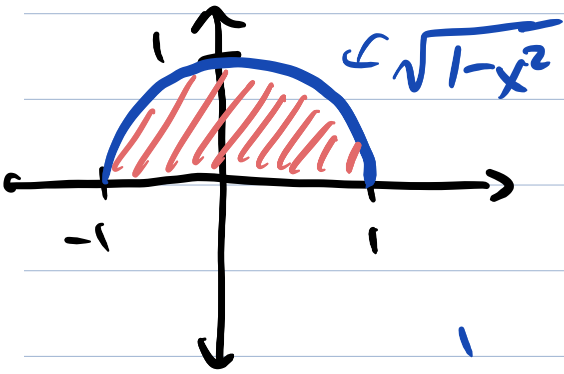
estimate: area of R1 + area of R2

$= \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{2}{3} = \frac{1}{2}+\frac{1}{3} = \boxed{\frac{5}{6}}$

≈ 0.83

Actual area $= \int_1^2 \frac{1}{t}\,dt = \ln(2) \approx 0.693$

Ex: The top half of a circle with radius 1 can be described by the function $y=\sqrt{1-x^2}$. What is $\int_{-1}^{1}\sqrt{1-x^2}\,dx$?

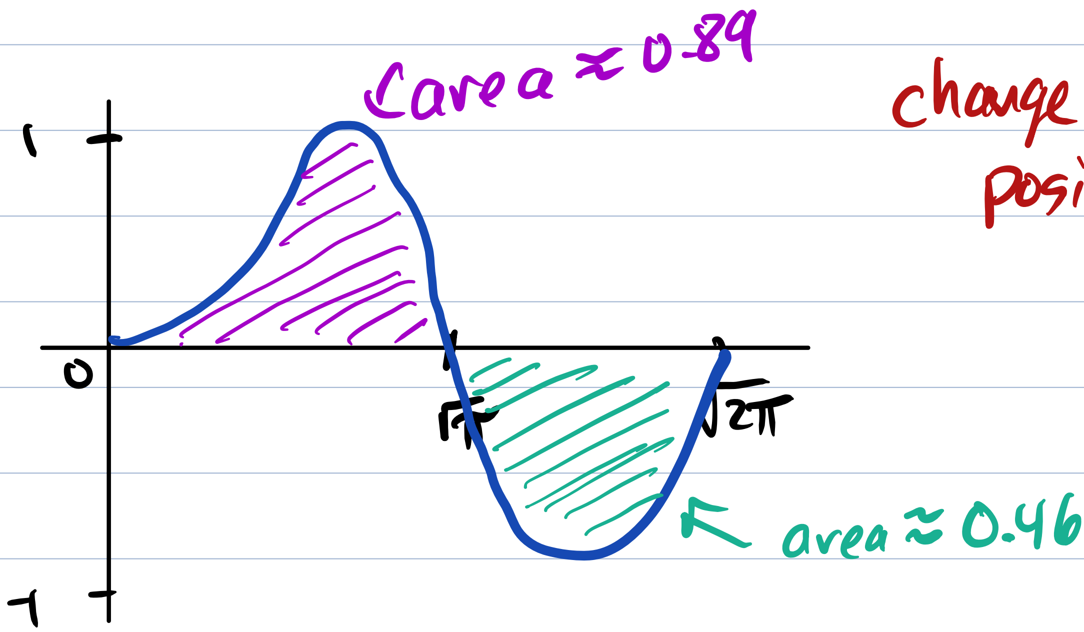


$$\int_{-1}^{1} \sqrt{1-x^2}\,dx = \text{half the area of a circle with radius } 1$$

$$= \frac{1}{2}\cdot\pi = \frac{\pi}{2} \approx 1.57$$

Remember that with integrals, area <u>under</u> the x-axis counts as negative.

$$\int_{0}^{\sqrt{2\pi}} \sin(\theta^2)\,d\theta \approx 0.89 - 0.46 = 0.43$$



change in position