Friday, Nov. 18 - Fall '22
Lecture #34

Announcements / Reminders

* Wiley Plus #12 due TUESDAY night (4.3)
* Wiley Plus #13 due the following Wednesday (4.6)

* Monday: Lecture
* Tuesday: Discussion
* Wed - Fri: Thanksgiving Break

} Help Desk + Office Hours → on Microsoft Teams

Wiley Plus due!

* Drop Deadline today! * ODS Email

Section 4.3 - Optimization and Modeling

Example (from the book)

What are the dimensions of an aluminum cylinder can that holds 40 in^3 of juice that uses the least material.

Constraint: volume = 40

Goal: minimize surface area

Volume $= \pi r^2 h$

Surf. Ar $= 2\pi r^2 + 2\pi r h$

Minimize $2\pi r^2 + 2\pi r h$ subject to

$$\pi r^2 h = 40.$$

Step 1) Use the constraint to solve for one variable in terms of the other

Step 2) Plug that into the function you're optimizing

Step 3) Find the global extrema

$\pi r^2 h = 40 \implies h = \frac{40}{\pi r^2}$ $\left(r = \sqrt{\frac{40}{\pi h}} \text{ messier} \right)$

$2\pi r^2 + 2\pi r h \implies 2\pi r^2 + 2\pi r\left(\frac{40}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{80}{r}$ (surface area)

Step 3: Find the global minimum of

$S(r) = 2\pi r^2 + \frac{80}{r}$

Find critical points and endpoints.

rewrite S: $S(r) = 2\pi r^2 + 80r^{-1}$

$$\Rightarrow S'(r) = 4\pi r + 80(-1)r^{-2}$$

$$= 4\pi r - \frac{80}{r^2}.$$

$S'(r) = 0$

$$\Rightarrow 4\pi r - \frac{80}{r^2} = 0$$

$$\Rightarrow 4\pi r = \frac{80}{r^2}$$

$$\Rightarrow 4\pi r^3 = 80$$

only critical point

$$\Rightarrow r^3 = \frac{20}{\pi} \Rightarrow r = \left(\frac{20}{\pi}\right)^{1/3}$$

$$\approx 1.85$$

$S'(r)$ undefined?

$S'(r)$ is undefined at $r=0$, but so is $S(r)$, so it doesn't count as a critical point.

Endpoints? What is the interval of possible r values and does it have endpoints?

$$(0, \infty)$$

no endpoints to check

The critical point $\left(\frac{20}{\pi}\right)^{1/3}$ is the only candidate to be the global minimum.

Let's do the S.D.T. to confirm it's a minimum:

$$S'(r) = 4\pi r - 80r^{-2}$$

$$S''(r) = 4\pi + \frac{160}{r^3}$$

$$S''\left(\left(\frac{20}{\pi}\right)^{1/3}\right) > 0$$

yes, it's a minimum

Now we know $r = \left(\frac{20}{\pi}\right)^{1/3}$ is the radius that makes the can with minimal surface area.

≈ 1.85 m

$h = \dfrac{40}{\pi r^2}$

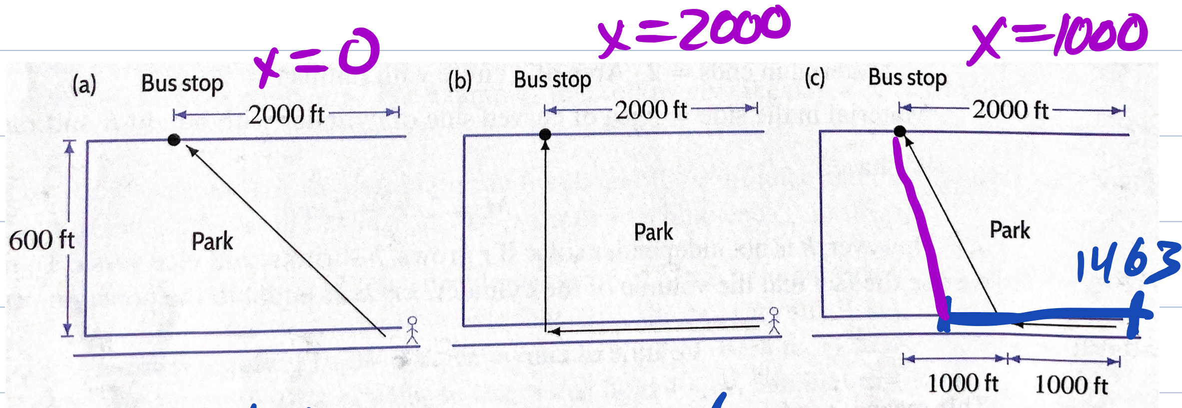
What's the corresponding height?

$$h = \frac{40}{\pi\left(\left(\frac{20}{\pi}\right)^{1/3}\right)^2} = \frac{40}{\pi\left(\frac{20}{\pi}\right)^{2/3}}$$

$$\approx 3.7 \text{ in}$$

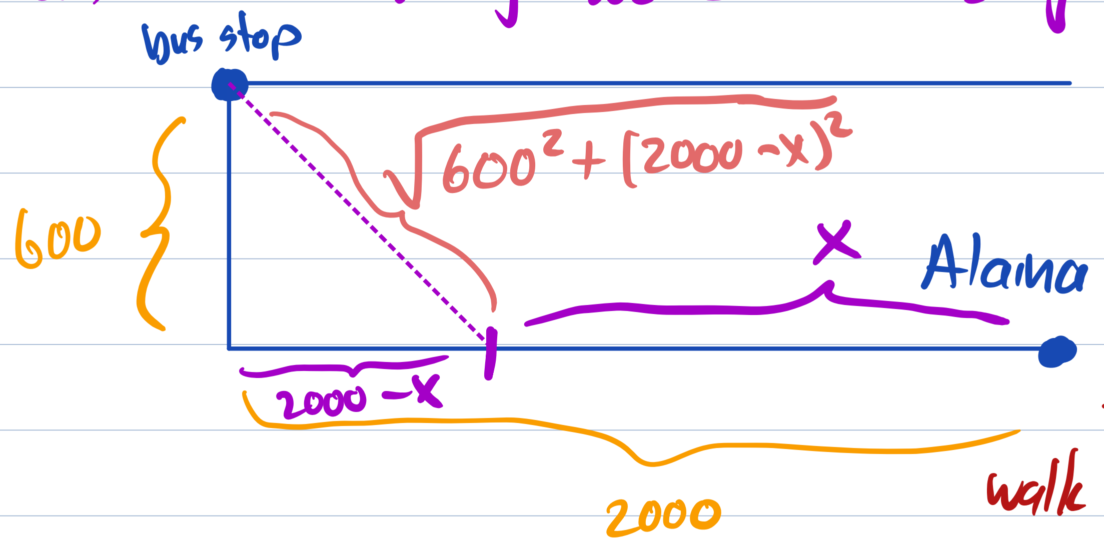
Ex:

Alaina wants to get to the bus stop as quickly as possible. The bus stop is across a grassy park, 2000 feet west and 600 feet north of her starting position. Alaina can walk west along the edge of the park on the sidewalk at a speed of 6 ft/sec. She can also travel through the grass in the park, but only at a rate of 4 ft/sec. What path gets her to the bus stop the fastest?



Let x be the distance walked on the sidewalk before going into the grass.

Constraint? Thing we're minimizing.



If Alaina walks x on the sidewalk, how far does she walk in the grass?

If Alana walks x feet on the sidewalk, then she must walk $\sqrt{600^2+(2000-x)^2}$ feet in the grass.

Sidewalk: 6 ft/sec
Grass: 4 ft/sec

If Alana walks x feet on the sidewalk HOW LONG does the whole journey take?

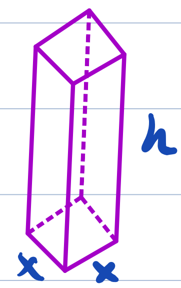
$$\boxed{\underbrace{\frac{x}{6}}_{\text{sidewalk time}} + \underbrace{\frac{\sqrt{600^2+(2000-x)^2}}{4}}_{\text{grass time}}}$$

Use 4.1 and 4.2 to find the global minimum.

The answer you get: $x = 2000 - 240\sqrt{5}$
≈ 1463 feet

Ex:

What is the maximum volume of a closed box with a square base and surface area 24 in^2?



Volume: x^2h

Surface Area: $2x^2+4xh$

Constraint: $2x^2+4xh=24$

Goal: Maximize x^2h

Step 1: Solve constraint for one variable.

$$h=\frac{24-2x^2}{4x}$$

Step 2: Plug this in to the expression we're maximizing

$$\text{volume}=x^2h=x^2\left(\frac{24-2x^2}{4x}\right)$$

$$=x\left(\frac{24-2x^2}{4}\right)$$

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$$V(x) = 6x - \frac{x^3}{2}$$

Step 3:

$$V'(x) = 6 - \frac{3}{2}x^2 = 0$$

$$\Rightarrow 6 = \frac{3}{2}x^2$$

$$\Rightarrow \frac{2}{3} \cdot 6 = \cancel{\frac{2}{3}} \cdot \cancel{\frac{3}{2}} x^2$$

$$\Rightarrow 4 = x^2 \Rightarrow x = 2$$

endpoints? No $(0, \infty)$

$$V''(x) = -3x$$

$$V''(2) = -6 < 0$$

S.D.T. $\Rightarrow$ $x = 2$ is a max

$$h = \frac{24 - 2x^2}{4x} = \frac{24 - 8}{8} = \frac{16}{8} = 2$$

$2 \times 2 \times 2$