Friday, Nov 11 - Fall '22
Lecture # 31 /41

Announcements / Reminders

* Wiley Plus #11 due next Wed (3.10, 4.1, 4.2, some 4.3)
* Quiz 10 next Thurs. (same)

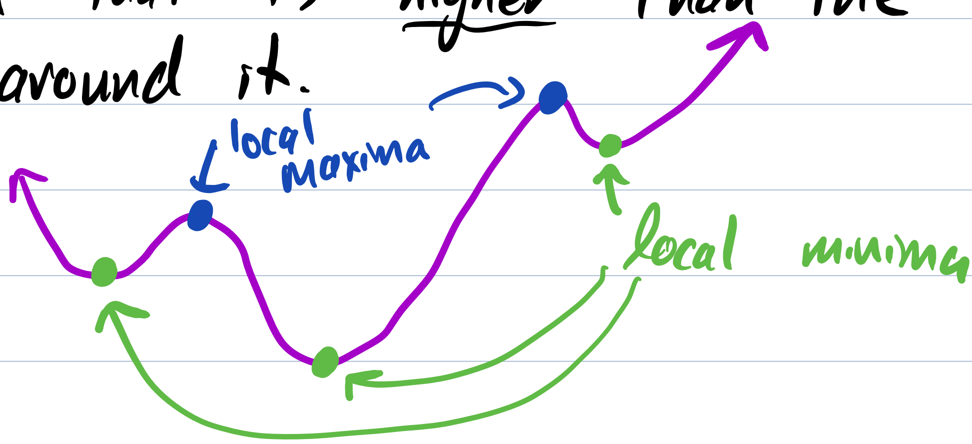
* Drop Deadline next Fri.

Continuing 4.1 - Using First and Second Derivatives

Local Maxima and Minima

"local" vs. "global"

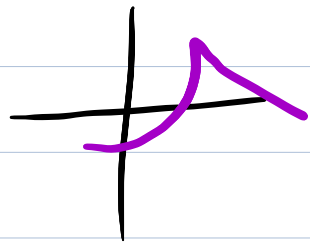
* A local minimum is a point on a function that is lower than the points around it.
* A local maximum is a point on a function that is higher than the points around it.

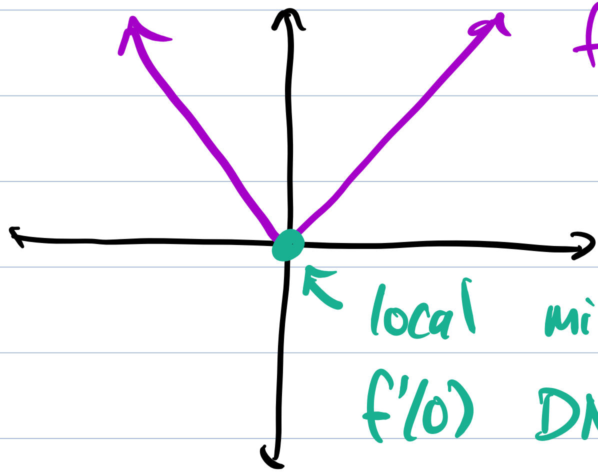


Finding local minima and maxima:

Idea When there's a local max or local min, the derivative must be zero (the function turns around).

This is a slight lie:





Def: A critical point is an x-value $x=p$ in the domain of $f(x)$ where either:

(1) $f'(p)=0$

OR

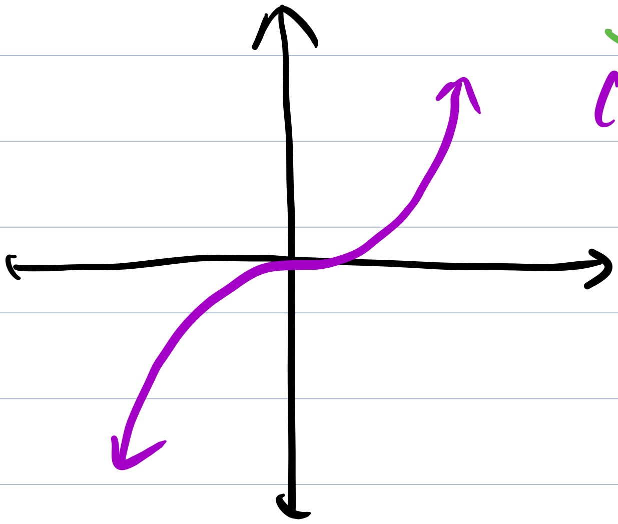
(2) $f'(p)$ is undefined

minima or maxima

Theorem: All local extrema happen at critical points.

Warning: Not all critical points are local extrema.

Ex: $f(x) = x^3$

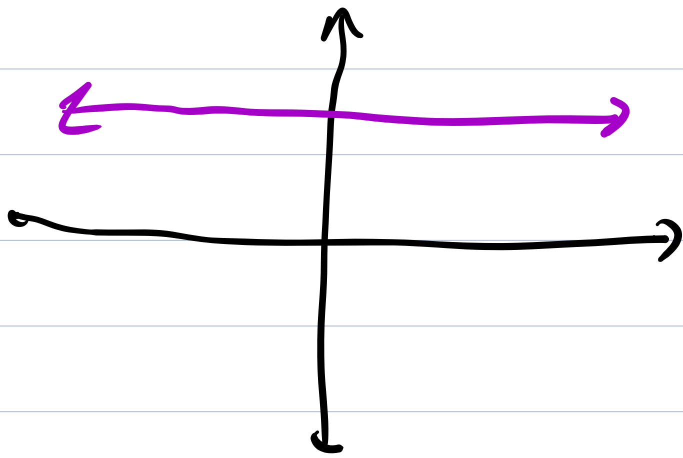


$x=0$

$(0,0)$ is a critical point because $f'(0) = 0$

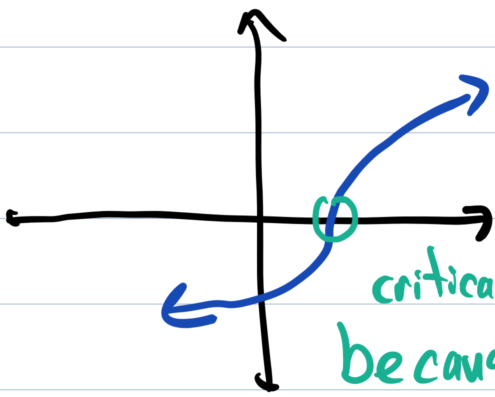
$(0,0)$ is neither a local min nor a local max

Ex: constant function / horizontal line



$f(x) = 2$ every # is a critical point

no local extrema.



critical point because f' DNE

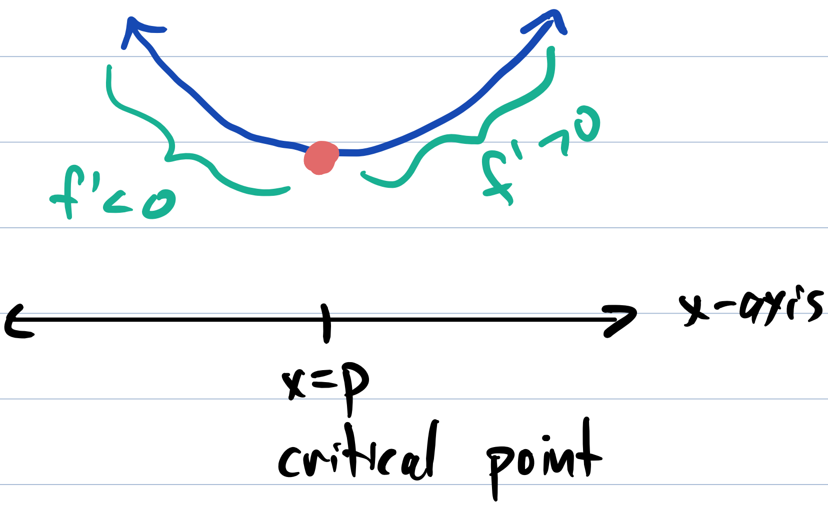
Algorithm for finding local extrema of $f(x)$:

(1) Find the critical points.

- compute $f'(x)$
- solve $f'(x)=0$
- find where f' is undefined

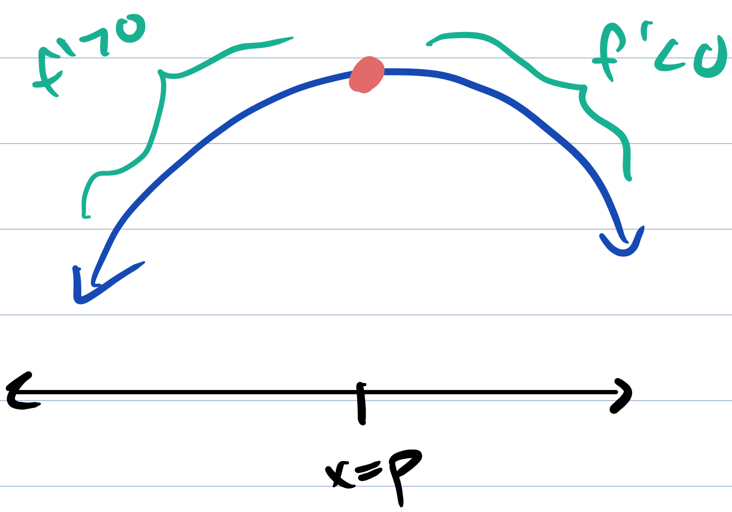
$\Big\}$ critical points

(2) Check each critical point to see if it's a local min, a local max, or neither.



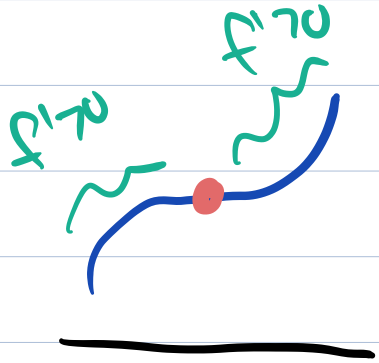
local min

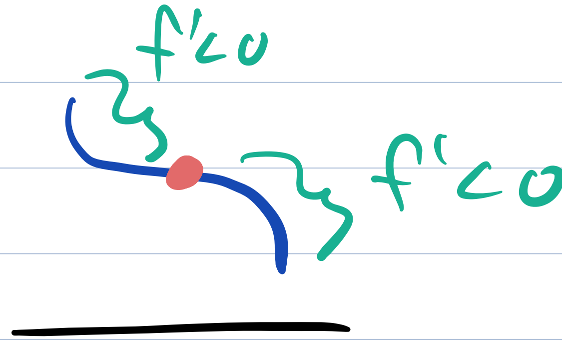
If $f'<0$ a little to the left and $f'>0$ a little to the right then it's a local min.



If $f'>0$ a little to the left and $f'<0$ a little to the right, then it's a local max.

Neither a max nor a min:





If f' is positive on both sides or negative on both sides, then there's __no__ local extremum.

__Ex:__ $f(x) = \dfrac{1}{x(x-1)}$ vertical asymptotes $x=0$ and $x=1$

Goal: Find all local extrema.

Step 1: $f'(x) = -\dfrac{2x-1}{(x^2-x)^2} = -\dfrac{2x-1}{(x(x-1))^2}$

$f'(x)=0 \Rightarrow -\dfrac{2x-1}{(x^2-x)^2} = 0 \Rightarrow -(2x-1)=0$

$\Rightarrow x = \frac{1}{2}$

$f'(x)$ DNE?

$\underbrace{x=0 \text{ and } x=1}_{\text{These are not in the domain of } f, \text{ so not critical points}}$

Critical points: $x = \frac{1}{2}$

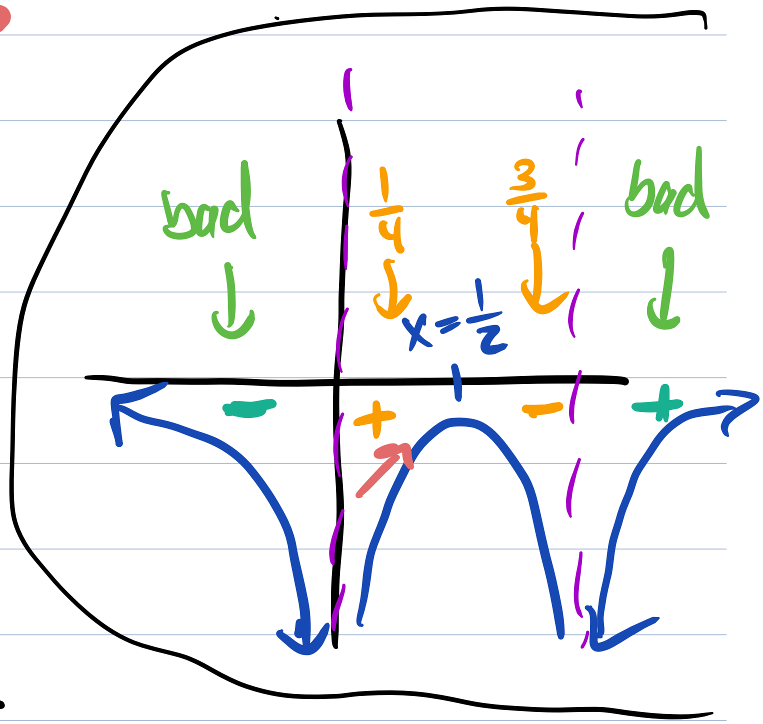
(2) First Derivative Test:
plug in values to f' a little to the left and right of $\frac{1}{2}$.

Don't want to skip past some other critical point or a place where the function isn't continuous

$-\dfrac{2x-1}{(x^2-x)^2}$

$f'\left(\frac{1}{4}\right) = - \cdot \frac{-}{+} = +$

$f'\left(\frac{3}{4}\right) = - \cdot \frac{+}{+} = -$



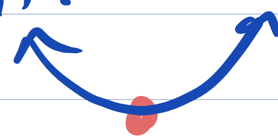
The function goes up to $x = \frac{1}{2}$, then down.

Thus, $x = \frac{1}{2}$ is a local maximum.

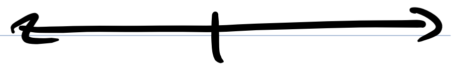
This is called the <u>First Derivative Test</u>.

Another possibility is the
<u>Second Derivative Test</u>

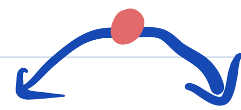
Suppose $x=p$ is a critical point.



* If $f''(p)>0$, then p is a local min.



* If $f''(p)<0$, then p is a local max.



* If $f''(p)=0$, then you can draw <u>no conclusion</u>.
(could be a max, could be a min, could be neither)