Wednesday, Nov 9 – Fall '22
Lecture #30

Announcements / Reminders

* Wiley Plus #10 due today (3.4, 3.5, 3.6, 3.9)
* Quiz 9 tomorrow (same)

* Reminder: Course Website: jaypantone.com
  click Math 1450
* EI bonus points entered – double check it!

Continuing Section 3.10 – Theorems about Differentiable Functions

The Racetrack Principle

If two cars start in the same spot, and Car 2 always goes faster than Car 1, then Car 2 will always be ahead of Car 1. (duh)

If two cars end in the same spot (at the same time), and Car 2 always goes faster than Car 1, then Car 2 was always <u>behind</u> Car 1. (duh)

Car 1's position Car 2's position

Mathematically: Suppose $g(x)$ and $h(x)$ are continuous and differentiable in $[a,b]$ and $g'(x) \le h'(x)$ everywhere in $[a,b]$.

Car 2 is always going faster (or the same).

We can conclude:

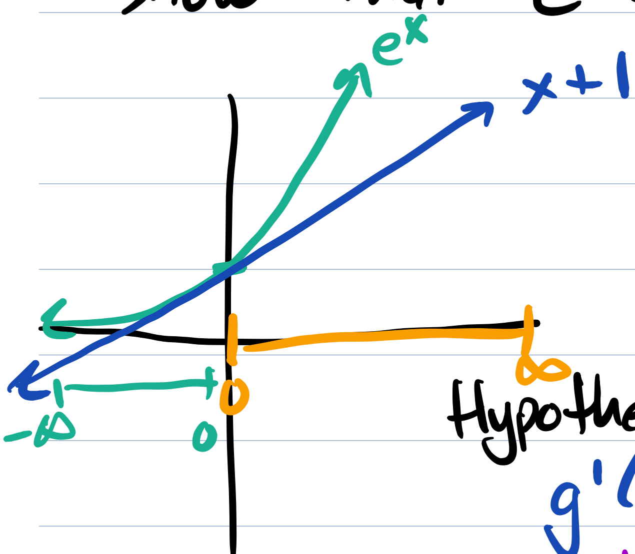
Car 2 is always ahead.

* If $g(a)=h(a)$, then $g(x) \le h(x)$ for all x in $[a,b]$

start in same spot

* If $g(b)=h(b)$, then $h(x) \le g(x)$ for all x in $[a,b]$.

<u>Ex</u>: The Racetrack Principle can be used to show that $e^x \ge x+1$ for all x.



Car 2 $= e^x$

$h(x) = e^x$

$h'(x) = e^x$

Car 1 $= x+1$

$g(x) = x+1$

$g'(x) = 1$

Hypotheses: g, h continuous ✓ diff'ble ✓

$g'(x) \le h'(x)$ everywhere ✗

$1 \le e^x$ for all x values

When $x \geq 0$: $g'(x) \leq h'(x)$

By the RTP: implies $g(x) \leq h(x)$ ✓

When $x \leq 0$: $g'(x) \geq h'(x)$

By the RTP: implies $g(x) \leq h(x)$ ✓

Chapter 4 – Solving Real Problems

Section 4.1 – Using First and Second Derivatives

In 4.1 and 4.2 we'll learn how to use derivatives to analyze functions.

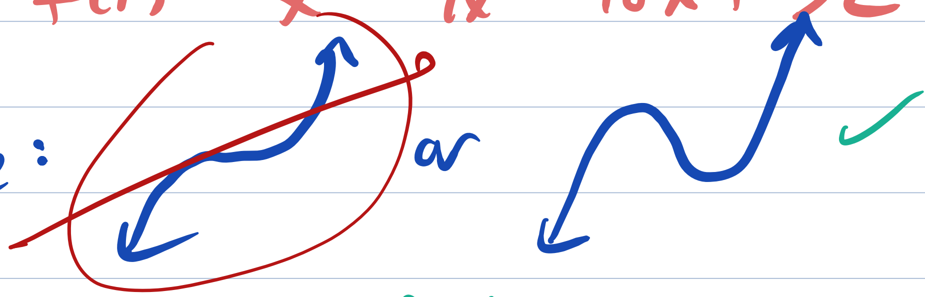
We've seen a lot already:

Summary

* If $f' > 0$, then f is increasing.
* If $f' < 0$, then f is decreasing.

* If $f'' > 0$, then f is concave up
"bending upward"
* If $f'' < 0$, then f is concave down

Ex: Analyze $f(x) = x^3 - 9x^2 - 48x + 52$

General Shape: 

$f'(x) = 3x^2 - 18x - 48$ factor

$= 3(x^2 - 6x - 16)$

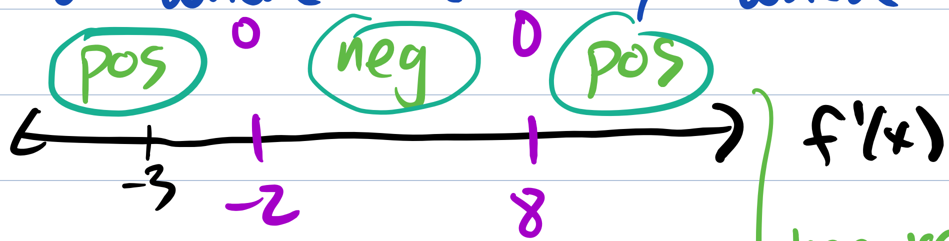
$= 3(x-8)(x+2)$

$f'(0) = -48$

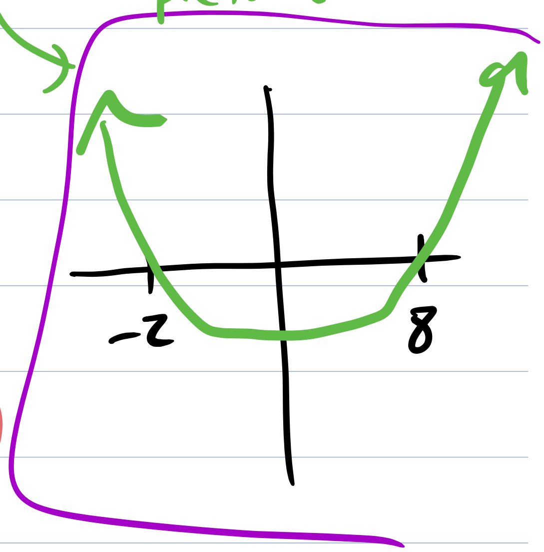
$f'(x) = 0$ when $x = 8$ and $x = -2$

$\Rightarrow f(x)$ is flat at $x = 8$ and $x = -2$

We know where $f'(x) = 0$, where is pos/neg?



because of the picture.



What's another way to know if f' is pos or neg on these three intervals?

Plug in a # in each interval and see if it's pos or neg.

$f'(-3) = 3 \cdot (-3-8) \cdot (-3+2)$

pos • neg • neg = pos

$f'(0) = 3 \cdot (0-8) \cdot (0+2)$ = pos · neg · pos = neg

$f'(10) = 3 \cdot (10-8) \cdot (10+2)$ = pos

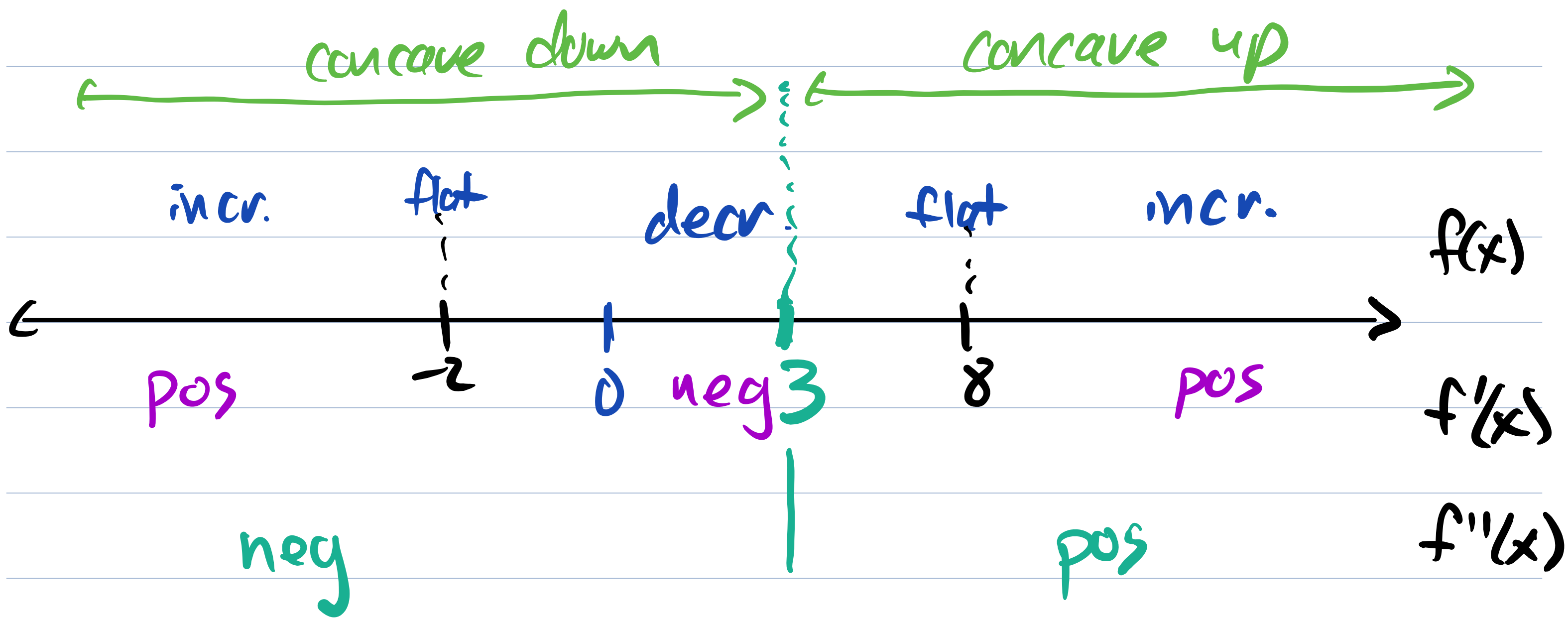
So far: $f(x) = x^3 - 9x^2 - 48x + 52$

$$f'(x) = 3x^2 - 18x - 48$$
$$= 3(x-8)(x+2)$$
$$f''(x) = 6x - 18 = 6\cdot(x-3)$$

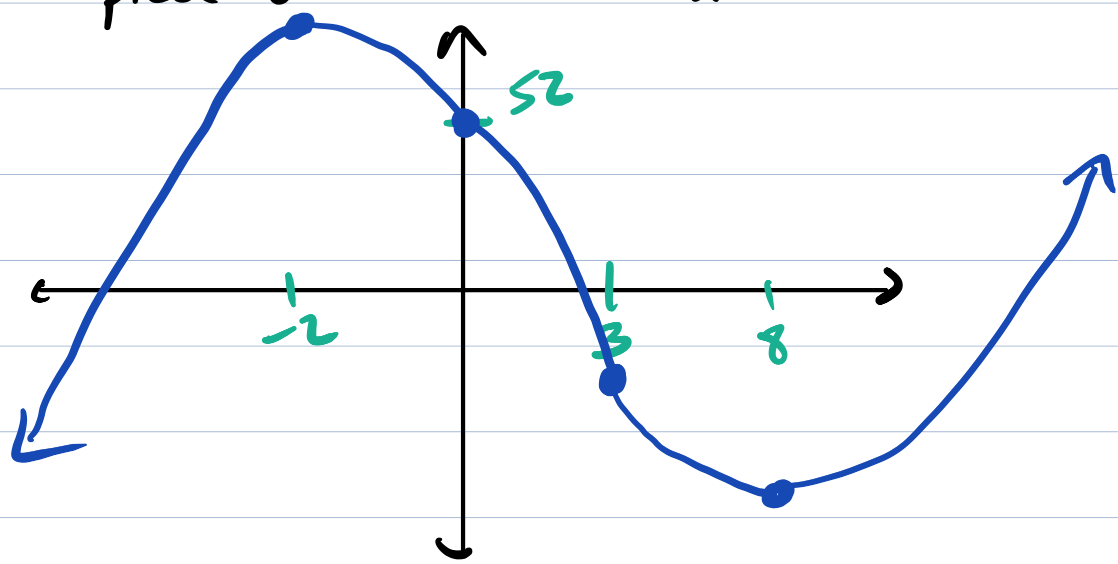
$f''(x) = 0$ at $x = 3$

$f''(0) = -18$ neg $\qquad$ $f''(5) = 30 - 18 =$ pos

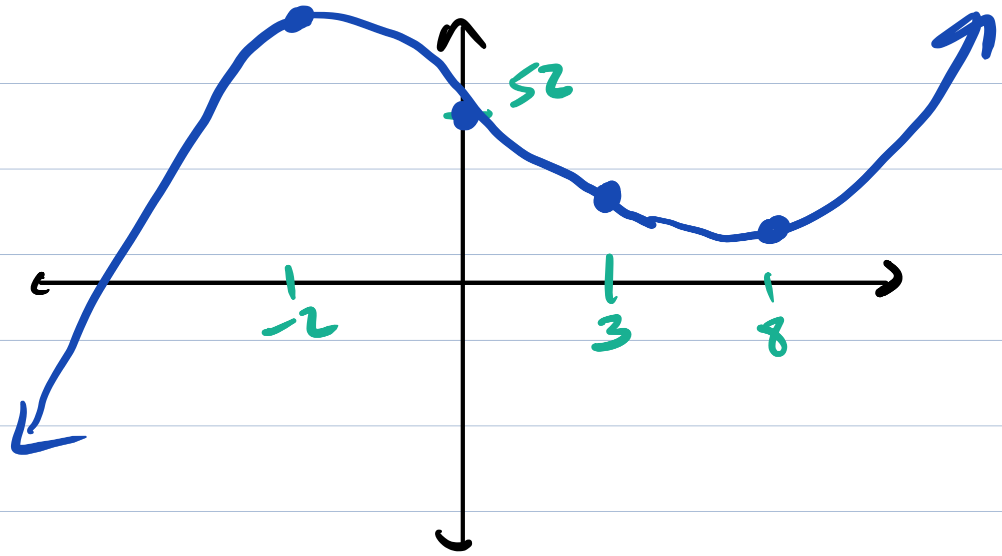
$\leftarrow \quad \rightarrow$



One more piece of information: $f(0) = 52$



OR:



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