Monday :(
~~Friday~~, Oct. 24 - Fall '22

Lecture #23

Announcements / Reminders

* Exam 2 in class on Wed
  (covers everything up to and including today)
* ODS proctoring
* Wiley Plus #8 due Wed night but you should really do it before the exam! (2.6, 3.1)
* Quiz 7 on Thursday (2.6, 3.1)
* Midterm Grades

3.1 – Powers and Polynomials

Rule 1: $$\frac{d}{dx}(c \cdot f(x)) = c \cdot f'(x)$$

$$(f(x)+g(x))'$$

Rule 2: $$\frac{d}{dx}(f(x)+g(x)) = f'(x)+g'(x)$$

$$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$$

Rule 3: $$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$$

WARNINGS:

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) \neq f'(x)\cdot g'(x)$$

$$\frac{d}{dx}\left(2^x\right) \neq x\cdot 2^{x-1}$$

The power rule doesn't work!

If this were true, it work when $f(x) = x$ and $g(x) = x$

$$\underbrace{\frac{d}{dx}\left(f(x)\cdot g(x)\right)} \neq f'(x)\cdot g'(x)$$

$$\frac{d}{dx}(x\cdot x) \qquad \frac{d}{dx}(x)\cdot\frac{d}{dx}(x)$$

$$= \qquad\qquad 1\cdot 1$$

$$\frac{d}{dx}(x^2) \qquad\qquad =$$

$$= $$

$$2x \neq 1$$

Let $f(x) = x^{10}$.

$$f'(x) = 10 \cdot x^9$$

$$f''(x) = 10 \cdot \frac{d}{dx}(x^9)$$

$$= 10 \cdot (9x^8) = 90x^8$$

$$\frac{d}{dx}(e^3) = 0$$

Just some #

Derivative of a polynomial
Just combine the three rules.

Ex: Find the derivative of $5x^2 + 3x + 2$.

$$\frac{d}{dx}(5x^2 + 3x + 2) = \frac{d}{dx}(5x^2) + \frac{d}{dx}(3x) + \underbrace{\frac{d}{dx}(2)}_{=0}$$

rule 2

$$= \underbrace{5\cdot\frac{d}{dx}(x^2) + 3\cdot\frac{d}{dx}(x)}_{\text{rule 1}} + 0$$

$$= 5\cdot(2x) + 3\cdot(1)$$

$$= 10x + 3$$

Ex: $\frac{d}{dx}\left(5\cdot\sqrt{x} - \frac{10}{x^2} + \frac{1}{2\sqrt{x}}\right)$

Recall: $\sqrt{x} = x^{1/2}$ $\qquad \frac{1}{\sqrt{x}} = x^{-1/2}$

$$\frac{1}{x^2} = x^{-2}$$

WARNING! $\frac{d}{dx}\left(\frac{1}{x^2}\right) \neq \frac{1}{2x}$

$$-2\cdot x^{-2-1}$$

$$= -2x^{-3} = \frac{-2}{x^3}$$

$$\frac{d}{dx}\left(5\cdot\sqrt{x} - \frac{10}{x^2} + \frac{1}{2\sqrt{x}}\right) \qquad \frac{1}{2}\cdot\frac{1}{\sqrt{x}}$$

$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \|$$

$$= \frac{d}{dx}(5\sqrt{x}) - \frac{d}{dx}\left(\frac{10}{x^2}\right) + \frac{d}{dx}\left(\frac{1}{2\sqrt{x}}\right)$$

$$= \frac{d}{dx}(5x^{1/2}) - \frac{d}{dx}(10x^{-2}) + \frac{d}{dx}\left(\frac{1}{2}\cdot x^{-1/2}\right)$$

$$= 5\cdot\frac{d}{dx}(x^{1/2}) - 10\frac{d}{dx}(x^{-2}) + \frac{1}{2}\cdot\frac{d}{dx}(x^{-1/2})$$

$$= 5\cdot\frac{1}{2}\cdot x^{\frac{1}{2}-1} - 10\cdot(-2)x^{-2-1} + \frac{1}{2}\cdot\left(-\frac{1}{2}\right)x^{-\frac{1}{2}-1}$$

$$= \frac{5}{2}x^{-\frac{1}{2}} + 20x^{-3} - \frac{1}{4}x^{-3/2}$$

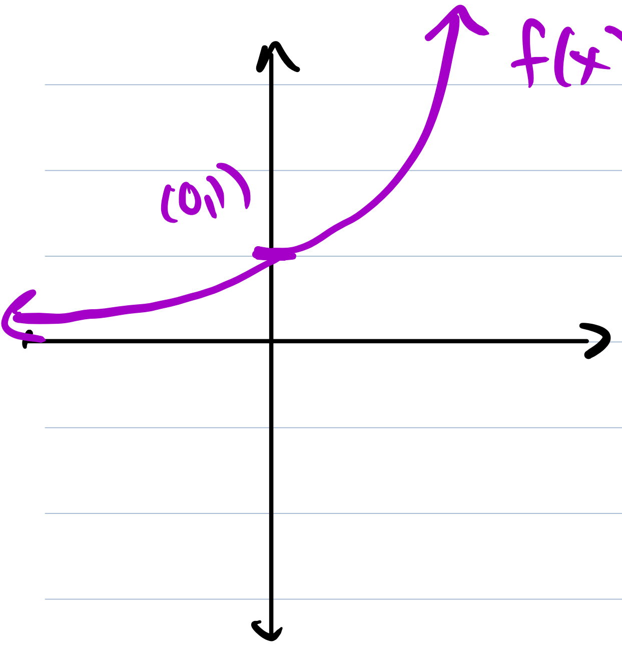
$$= \frac{5}{2\sqrt{x}} + \frac{20}{x^3} - \frac{1}{4x^{3/2}}$$

3.2 - The Exponential Functions

What is the derivative of 2^x?

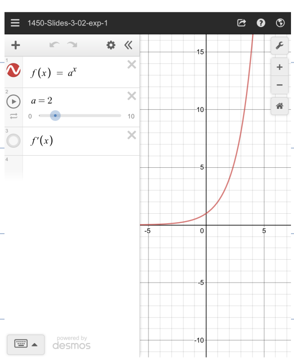
In 3.1: The derivative of a degree 4 polynomial is a degree 3 polynomial.
The der. of a deg. 5 poly. is a deg. 4 poly and so on.

$f(x) = a^x, \quad a > 1$



Properties of f':

* always > 0
* always increasing
* $\lim_{x \to -\infty} f'(x) = 0$
* $\lim_{x \to \infty} f'(x) = \infty$



$$a^{b+c} = a^b \cdot a^c$$

Let $g(x) = 2^x$.

$$2^x \cdot 2^h$$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h \to 0} \left[2^x \left(\frac{2^h - 1}{h} \right) \right]$$

$$= 2^x \cdot \underbrace{\lim_{h \to 0} \left(\frac{2^h - 1}{h} \right)}$$

No x, so this limit equals just some # c.

$$= c \cdot 2^x$$

$\Rightarrow$ The derivative of 2^x is a constant times 2^x.

$$\boxed{\underline{\text{Formula:}} \quad \frac{d}{dx}(a^x) = \ln(a) \cdot a^x}$$

Nice case:

$$\frac{d}{dx}(e^x) = \ln(e)\cdot e^x = 1\cdot e^x = e^x$$

exponential

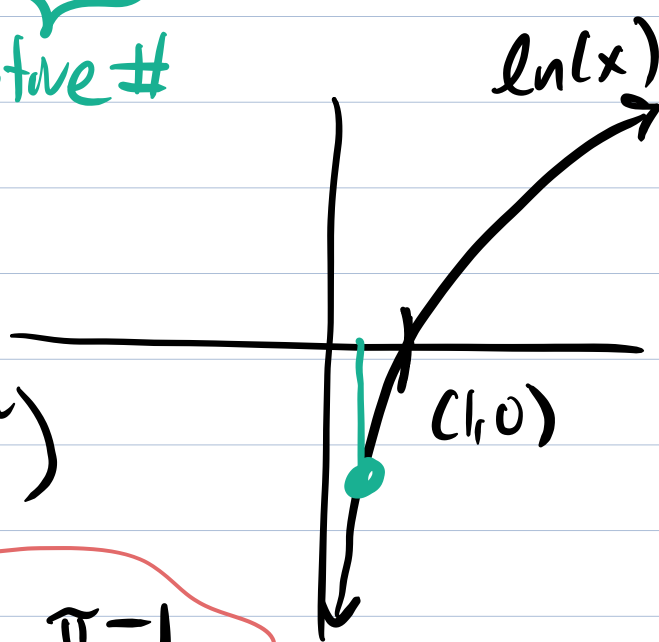
$$\frac{d}{dx}(e^3) = 0$$

constant

power rule

$$\frac{d}{dx}(x^e) = e\cdot x^{e-1}$$

Ex: $\frac{d}{dx}\left(\left(\frac{1}{2}\right)^x\right) = \underbrace{\ln\left(\frac{1}{2}\right)}_{\text{negative \#}}\cdot\left(\frac{1}{2}\right)^x$



Ex: $\frac{d}{dx}\left(2\cdot 3^x + x^\pi\right)$

$$= \frac{d}{dx}(2\cdot 3^x) + \frac{d}{dx}(x^\pi)$$

$$= 2\cdot\ln(3)\cdot 3^x + \pi\cdot x^{\pi-1}$$