Wednesday, Oct. 19 - Fall '22
Lecture #22

Announcements / Reminders

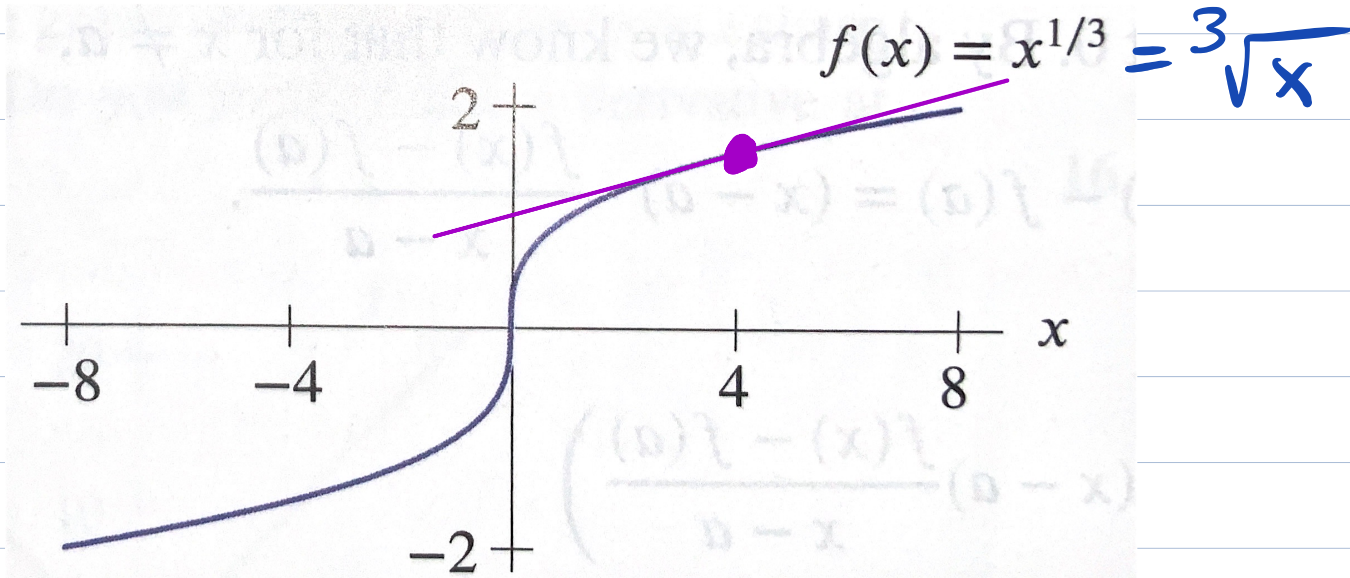
* Wiley Plus #7 tonight (2.3, 2.4, 2.5)
* Wiley Plus #8 available tomorrow morning (2.6, 3.1)
* No discussion Thurs, no lecture Fri! (so no quiz this week)
* Exam 2 is Wed, Oct 26
  covers material from Fri, Sept 30 to Monday, Oct 24
* Read your email about midterm grades
* ODS Accommodations for Exam 2

Section 2.6 - Differentiability

What could make a function $f(x)$ **NOT** diff'ble at $x=a$?

1) if $f(a)$ doesn't exist
2) if f is not continuous at $x=a$
3) f has a sharp corner at $x=a$

4) f has a vertical TL at $x=a$



Is not differentiable at $x=0$

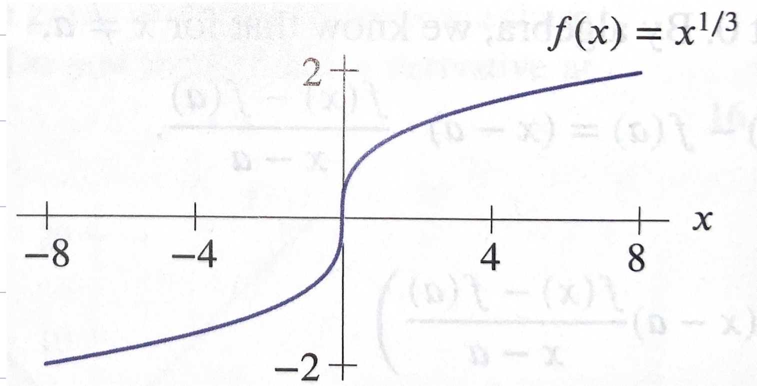
Differentiability vs. Continuity

Theorem (fact): If $f(x)$ is differentiable at $x=a$, then $f(x)$ must also be continuous at $x=a$.

$$\text{Diff'ble} \overset{(\text{"implies"})}{\Longrightarrow} \text{Continuous}$$

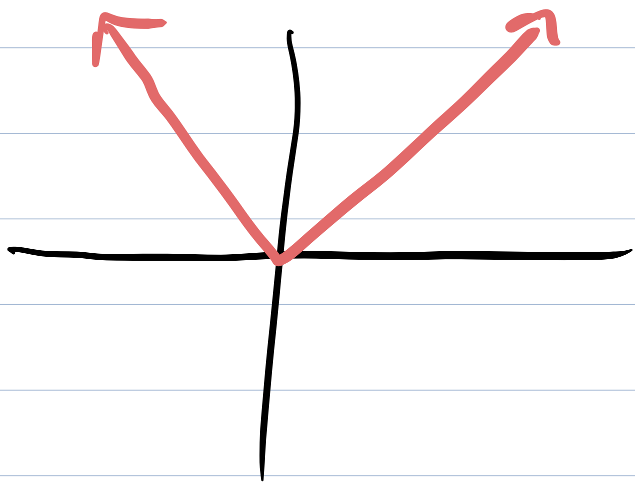
Is it true in reverse? In other words, does continuous $\Rightarrow$ diff'ble?

No. For example $x^{1/3}$.



This is continuous at $x=0$.
But not diff'ble.

Another example:



This is continuous at $x=0$.
But not diff'ble.

There's a proof of the theorem in the book.

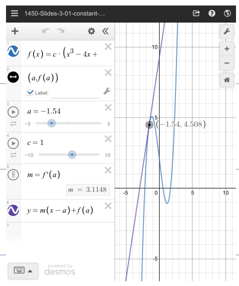
Chapter 3: Shortcuts to compute derivatives
Applications of Calculus

Section 3.1 – Powers and Polynomials

Topic 1: Constant Multiples

Let $f(x)$ be a function with derivative $f'(x)$.

What is the derivative of $c \cdot f(x)$?



<u>Theorem 3.1:</u> The derivative of $c \cdot f(x)$ is $c \cdot f'(x)$.

In our Leibniz notation:

$$\frac{d}{dx}\left(c \cdot f(x)\right) = c \cdot f'(x)$$

Even more notation:

$$\left(c \cdot f(x)\right)' = c \cdot f'(x)$$

Why is this true? Define $g(x)$ to equal $c \cdot f(x)$. We want to know $g'(x)$. By the limit definition of derivative:

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

⑤

$$= \lim_{h\to 0} \frac{(c\cdot f(x+h)) - c\cdot f(x)}{h}$$

$$= \lim_{h\to 0} \left(c \cdot \frac{f(x+h)-f(x)}{h}\right)$$

$$= c \cdot \lim_{h\to 0} \left(\frac{f(x+h)-f(x)}{h}\right)$$

$$= c\cdot f'(x).$$

Topic 2: Sums + Differences of Functions

Let $f(x)$ and $g(x)$ be functions.
What is the derivative of $f(x)+g(x)$?

Theorem 3.2: The derivative of $f(x)+g(x)$ is:

$$f'(x)+g'(x).$$

Other notation: $\frac{d}{dx}\left(f(x)+g(x)\right) = f'(x)+g'(x).$

WARNING: $\cancel{\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f'(x)\cdot g'(x)}$

Subtraction does work:

$$\frac{d}{dx}\left(f(x)-g(x)\right) = \frac{d}{dx}\left(f(x)+(-1)\cdot g(x)\right)$$

$$= \frac{d}{dx}\left(f(x)\right) + \frac{d}{dx}\left((-1)\cdot g(x)\right)$$

(by Theorem 3.2)

$$= \frac{d}{dx}\left(f(x)\right) + (-1)\cdot\frac{d}{dx}\left(g(x)\right)$$

(by Theorem 3.1)

$$= f'(x) + (-1)\cdot g'(x)$$

$$= f'(x) - g'(x)$$

Topic 3: Power Rule (again)

$$\frac{d}{dx}\left(x^n\right) = n\cdot x^{n-1}$$

Examples: $\frac{d}{dx}\left(x^3\right) = 3x^{3-1} = 3x^2$

$$\frac{d}{dx}\left(x^{-2/3}\right) = -\frac{2}{3}\cdot x^{-\frac{2}{3}-\frac{1}{1}}$$

$$-\frac{2}{3}-1=-\frac{2}{3}-\frac{3}{3}=\frac{-2-3}{3}=\frac{-5}{3}$$

common denominator

$$-\frac{2}{3}\cdot x^{-5/3}$$

Ex:

$$\frac{d}{dx}\left(-\frac{2}{3}x^{-5/3}\right)=-\frac{2}{3}\cdot\underbrace{\frac{d}{dx}\left(x^{-5/3}\right)}_{\text{power rule}}$$

$$=-\frac{2}{3}\cdot\left(-\frac{5}{3}x^{-8/3}\right)$$

$$=\frac{10}{9}x^{-8/3}$$

Non-example: Power rule only works when the base is a variable and the exponent is a # (no variables)

$$\cancel{\frac{d}{dx}\left(3^x\right)\neq x\cdot 3^{x-1}} \qquad \cancel{\frac{d}{dx}\left(x^x\right)\neq x\cdot x^{x-1}}$$