Fri, Oct. 7 - Fall '22
Lecture #17

Announcements / Reminders

* Wiley Plus #6 due next Wed (2.1, 2.2)
* Quiz #6 next Thurs (2.1, 2.2)
* Don't forget to check that your grades on D2L are correct!
* I will post a full exam answer key once makeups are done.

Section 2.2 - The derivative at a point

2.1: Average Rate of Change (Velocity) from $x=a$ to $x=b$:

$$\frac{\Delta \text{ value}}{\Delta \text{ time}} = \frac{f(b)-f(a)}{b-a}$$

"Δ" = change

Average RoC over a small window
$x=a \rightarrow x=a+h$ (h is small)

$$\frac{f(a+h)-f(a)}{h}$$

Instantaneous RoC at $x=a$

$$\lim_{h\to 0}\left[\text{the average RoC from } x=a \text{ to } x=a+h\right)$$

$$=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

We call this the "derivative of $f(x)$ at $x=a$" and use the notation

"$f'(a)$"

Summary:

[derivative of $f(x)$ at $x=a$]

= [instantaneous RoC of $f(x)$ at $x=a$]

$= f'(a)$ "f prime of a" ③

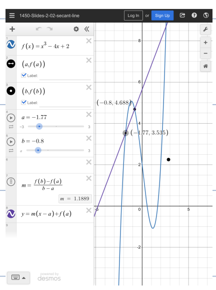
$$= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Slopes:

If you draw a line between $(a, f(a))$ and $(b, f(b))$ what is the slope of that line?

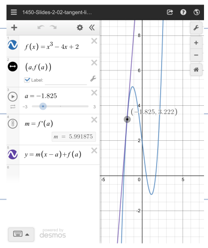
$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{f(b) - f(a)}{b - a}$$

= average RoC of $f(x)$ from $x = a$ to $x = b$



Secant Lines: Lines connecting two points on a graph.

The instantaneous RoC $f'(a)$ is the slope of the curve at $x=a$.



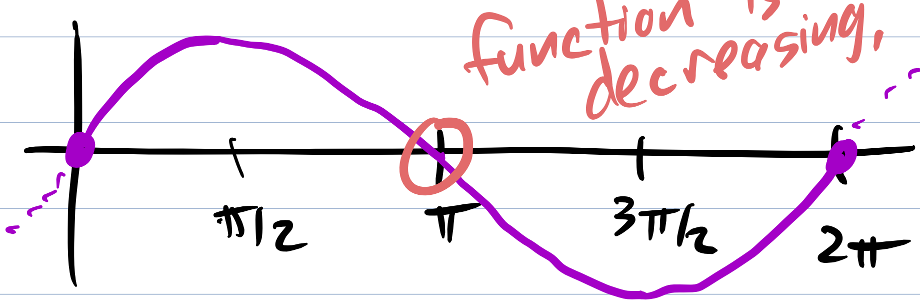
The tangent line at $x=a$ is the line whose slope is $f'(a)$ and passes through $(a, f(a))$.

$f'(a) > 0$: graph is increasing (going up) at $x=a$

$f'(a) < 0$: graph is decreasing at $x=a$

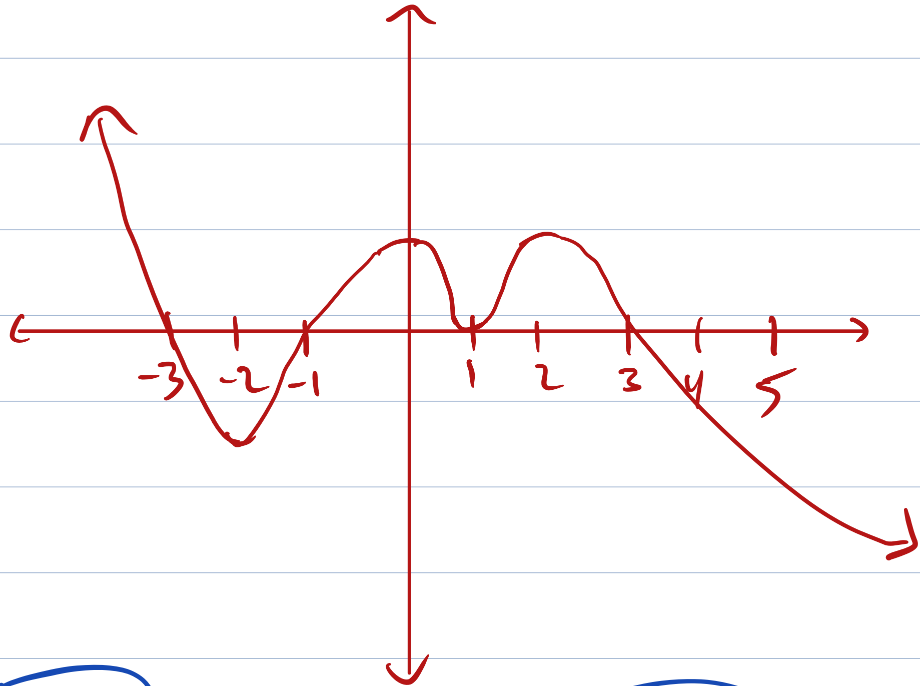
$f'(a) = 0$: graph is flat at $x=a$

Ex: Is the derivative of $\sin(x)$ positive, negative, or zero at $x=\pi$?



function is decreasing, so the deriv. is neg.

Group Work: On what intervals / points is the deriv. of the function pos., neg. or zero. ⑤



Pos : $(-2, 0)$, $(1, 2)$

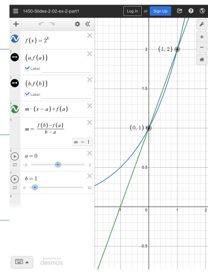
Negative : $(-\infty, -2)$, $(0, 1)$, $(2, \infty)$

Zero : $x = -2, 0, 1, 2$

Ex: Estimate the derivative of $f(x) = 2^x$ at $x = 0$.

One way: to compute average RoC from $x=0$ to $x=h$ where h is really small

Graphing Calc



Estimate is 0.69315.... ✓

good estimate

Ex: Find the equation for the tangent line of $f(x) = 2^x$ at $x = 0$ using the approx. above.

Formula for a line with slope m that passes through (a, b) is:

$$y = m \cdot (x - a) + b$$

$$y - b = m \cdot (x - a)$$

$m = 0.693$

point $(0, f(0)) = (0, 1)$

$$\boxed{y = 0.693 \cdot x + 1}$$ ✓