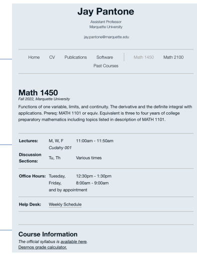
Fri, Sept. 30 - Fall '22
Lecture #14

Announcements / Reminders

* Wiley Plus #5 due Wed (1.8 and 1.9)
* Not finished grading exams, but no one should panic!

Grade Calculator



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Home CV Publications Software | Math 1450 Math 2100
Past Courses

Math 1450
Fall 2022, Marquette University
Functions of one variable, limits, and continuity. The derivative and the definite integral with applications. Prereq: MATH 1101 or equiv. Equivalent is three to four years of college preparatory mathematics including topics listed in description of MATH 1101.

Lectures:	M, W, F	11:00am - 11:50am
	Cudahy 001	
Discussion Sections:	Tu, Th	Various times
Office Hours:	Tuesday,	12:30pm - 1:30pm
	Friday,	8:00am - 9:00am
	and by appointment	
Help Desk:	Weekly Schedule	

Course Information
The official syllabus is available here.
Desmos grade calculator.

(Homework is worth 2.5 midterms)

Properties of Limits:

Assume $\lim\limits_{x \to c} f(x)$ and $\lim\limits_{x \to c} g(x)$ both exist and are finite.

(1) $\lim\limits_{x \to c} \left(b \cdot f(x)\right) = b \cdot \left(\lim\limits_{x \to c} f(x)\right)$

(2) $\lim\limits_{x \to c} \left(f(x) + g(x)\right) = \left(\lim\limits_{x \to c} f(x)\right) + \left(\lim\limits_{x \to c} g(x)\right)$

(3) $\lim\limits_{x \to c} \left(f(x) \cdot g(x)\right) = \left(\lim\limits_{x \to c} f(x)\right) \cdot \left(\lim\limits_{x \to c} g(x)\right)$

(2)

(4) $\lim_{x \to c} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$

as long as $\lim_{x \to c} g(x) \neq 0$

(5) $\lim_{x \to c} p(x) = p(c)$

for any polynomial $p(x)$

(because polynomials are continuous!)

Properties of Continuity

If $f(x)$ and $g(x)$ are continuous on some interval and b is any constant, then the following functions are also continuous on the same interval.

* $b \cdot f(x)$
* $f(x) + g(x)$
* $f(x) \cdot g(x)$
* $\frac{f(x)}{g(x)}$ as long as $g(x) \neq 0$ anywhere in the interval.

Two more facts:

(1) If $f(x)$ and $g(x)$ are continuous everywhere in their domain, then

$$f(g(x)) \quad \text{and} \quad g(f(x))$$

are also continuous wherever they exist.

(2) If $f(x)$ is continuous and invertible, then $f^{-1}(x)$ is continuous.

1.9 - Further Limit Calculations Using Algebra

In this section, we'll focus on limits of functions of the form $\dfrac{f(x)}{g(x)}$ at $x = \pm\infty$ and at points where $g(x) = 0$.

(1) $\displaystyle\lim_{x \to \infty} \frac{f(x)}{g(x)}$ and $\displaystyle\lim_{x \to -\infty} \frac{f(x)}{g(x)}$

If $g(x)$ "grows faster" than $f(x)$ as $x \to \infty$ or $x \to -\infty$, then

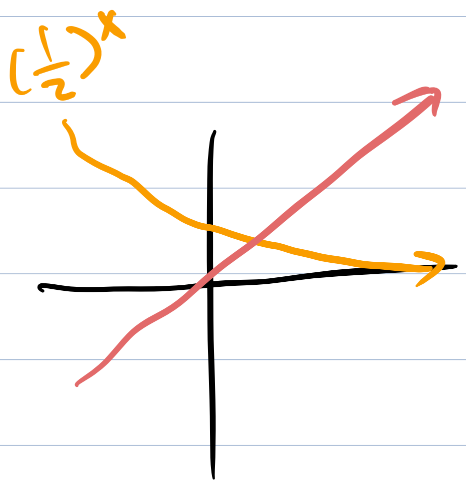
$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0 \quad \text{and} \quad \lim_{x\to -\infty} \frac{f(x)}{g(x)} = 0.$$ 4

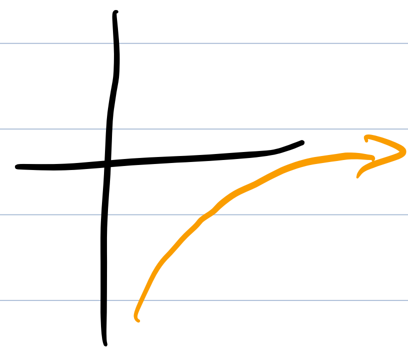
Exs: Which grows faster as $x \to \infty$?

$\frac{x^3}{-x^6}$

x^6	vs.	x^3
$-x^6$	vs.	x^3
x^3+2x+1	vs.	$1+x+x^4$
$x^{\frac{1}{2}}$ $\quad \sqrt{x}$	vs.	x $\quad x^1$
$5x^2$	vs.	$2x^2$
3^x	vs.	x^{100}
$\left(\frac{1}{2}\right)^x$	vs.	x

$$\frac{5\cancel{x^2}}{2\cancel{x^2}} = \frac{5}{2}$$





Exs: Which grows faster as $x \to -\infty$?

x^6	vs.	x^3

$-x^6$ vs. x^3

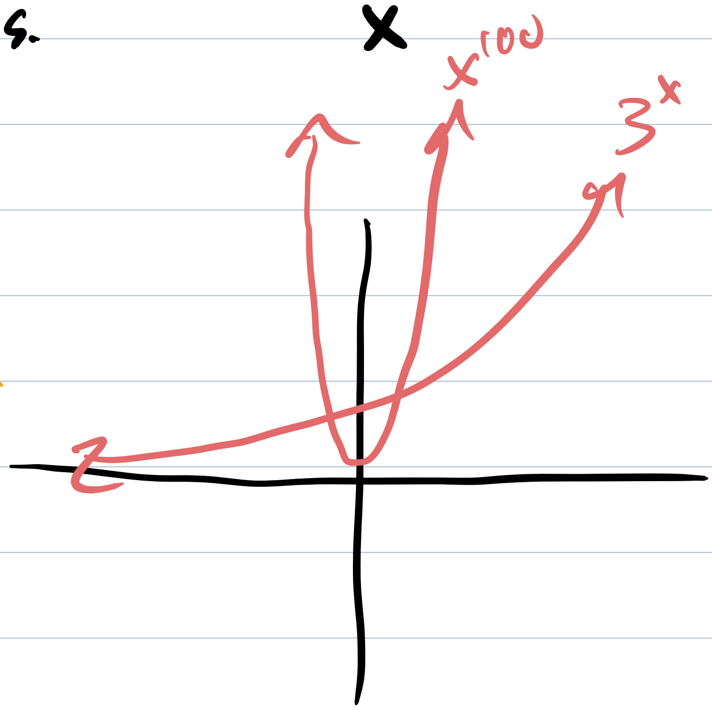
x^3+2x+1 vs. $1+x+x^4$

~~$\sqrt{x}$ vs. x~~ $\sqrt{x}$ doesn't exist as $x\to-\infty$

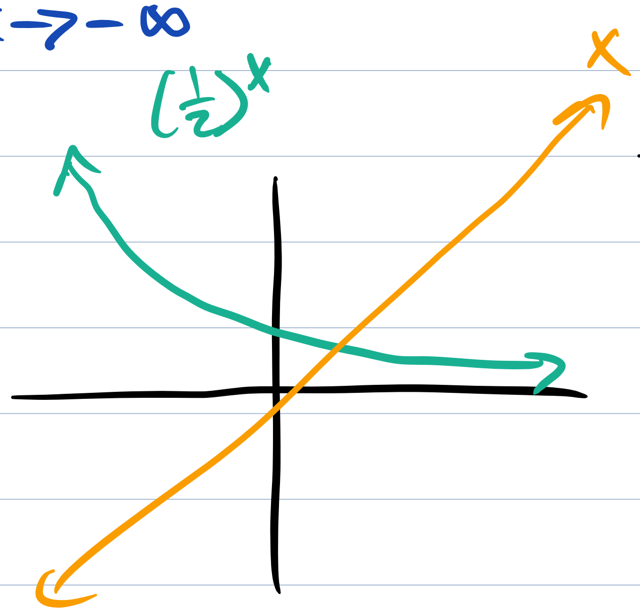
$5x^2$ vs. $2x^2$

3^x vs. x^{100}

$\left(\frac{1}{2}\right)^x$ vs. x



$\lim_{x\to-\infty} \frac{x}{\sqrt{x}}$ DNE



Some limit examples:

$$\lim_{x\to\infty} \frac{100x^3+3x+5}{0.0001x^6+1} = 0$$

" x^6 grows faster than x^3 "

$$\lim_{x \to \infty} \frac{x^{100} + x^{10}}{2^x + 1} = 0$$

$$\lim_{x \to -\infty} \frac{x^{100} + x^{10}}{2^x + 1} = +\infty$$

positive

positive

$2^{-BN} + 1$

(2) If $f(x)$ and $g(x)$ grow "equally fast", then you eliminate the "slower" terms and look at what's left.

$$\lim_{x \to \infty} \frac{5x^2 - \cancel{3x} + \cancel{1}}{2x^2 + \cancel{17}}$$

$5x^2$ vs. $2x^2$
equally fast

$$= \lim_{x \to \infty} \frac{5\cancel{x^2}}{2\cancel{x^2}} = \lim_{x \to \infty} \frac{5}{2} = \frac{5}{2}$$

Same as $x \to -\infty$.

* Can only do this when $x \to \pm\infty$.