Wed, Sept. 26 - Fall '22
Lecture #12

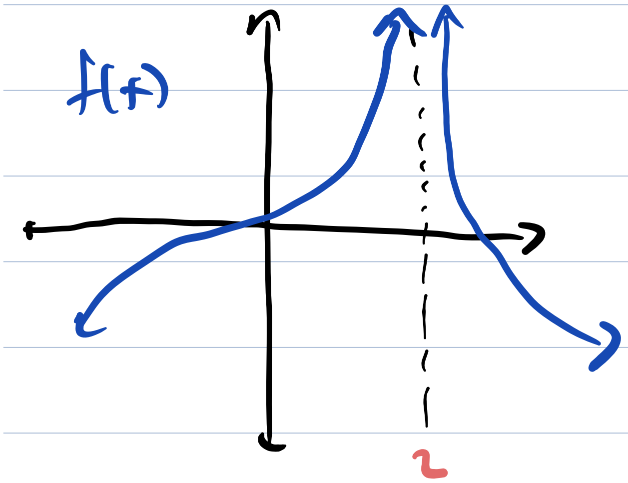
Announcements / Reminders

* Today: 1.7 + some of 1.8
* Tuesday: Discussion + Office Hours 12:30 – 1:30
* Wednesday: Exam 1 in class
  covers all material up to and including Mon, Sept. 26
  Wiley Plus HW 4 due at 11:59pm
  (1.6, 1.7, some of 1.8)
* Thursday: Discussion
  Quiz 4 – 1.7, some of 1.8
* Friday: Lecture

Exam Procedures:

→ Phones on silent, no headphones or smartwatches
→ No scratch paper or papers of any kind, just pen/pencil

Warning: The book and Wiley Plus are not always consistent with limits that are $\pm\infty$.



Sometimes they will say

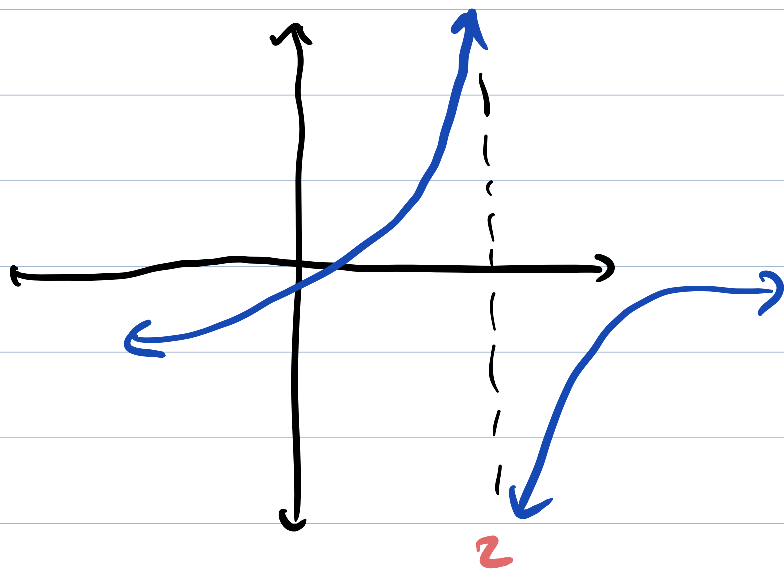
$$\lim_{x \to 2} f(x) = \text{DNE}$$

"na"

Sometimes they say

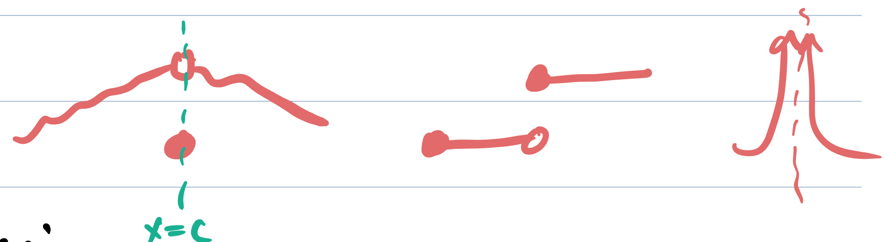
$$\lim_{x \to 2} f(x) = \infty$$ (better answer)

Not ambiguous:



$\lim_{x \to 2} f(x)$ really is "DNE"

Continuity Again



More precise definition:

The function $f(x)$ is continuous at $x=c$

if: (1) $f(c)$ exists

(2) $\lim_{x \to c} f(x) = f(c)$

"The value at c is exactly what the nearby points suggest it should be."

<u>Calculating Limits</u> many techniques depending on the function

$$\lim_{x \to c} f(x) = L$$

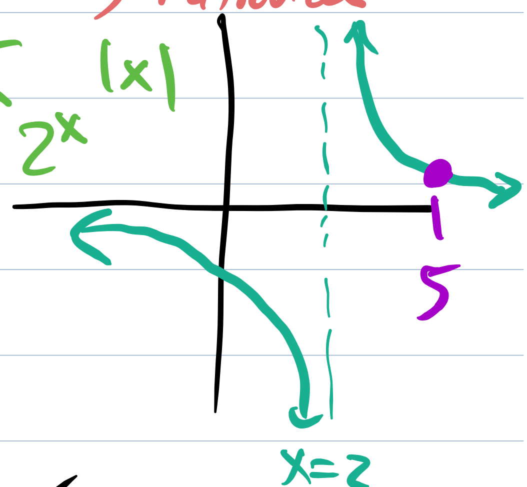
(1) If $f(x)$ is continuous at $x=c$, then the limit is just $f(c)$ (just plug in c)

$\frac{p(x)}{q(x)}$ p, q are poly. $\Rightarrow$ rational

$\sqrt{x}$ $|x|$

2^x

Ex: $\lim_{x \to 5} \frac{1}{x-2}$



$\frac{1}{x-2}$ is continuous at $x=5$ (everywhere except 2)

So plug in $x=5$:

$$\lim_{x \to 5} \frac{1}{x-2} = \frac{1}{5-2} = \frac{1}{3} \checkmark$$

(2) Sometimes we can do some algebra and some canceling.

$$\lim_{x \to 3} \frac{x^2-9}{x-3}$$

When you plug in $x=3$, you get 0 in the denominator, so the function is <u>NOT</u> continuous at $x=3$.

$$\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}}$$

$$= \lim_{x \to 3} (x+3)$$ plug in $x=3$

$$= 6$$

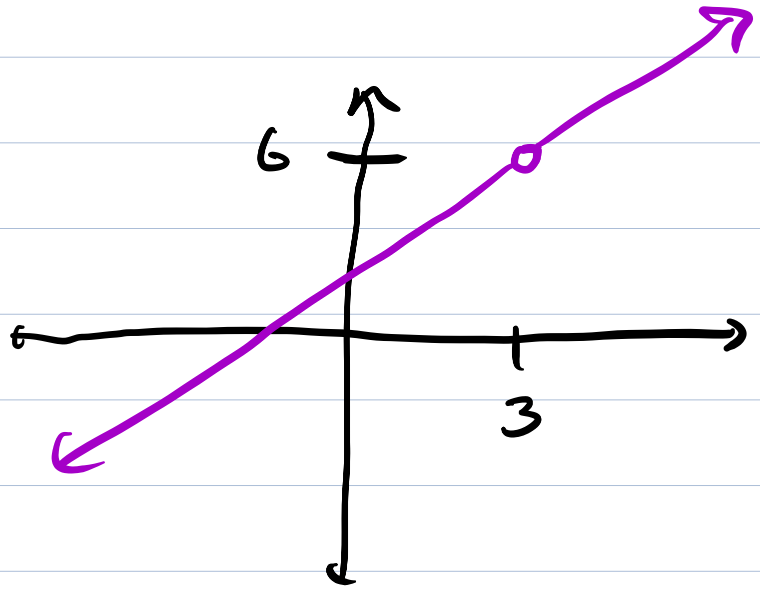
Side note:

$$\frac{(x-3)(x+3)}{(x-3)} = x+3$$

True for <u>every</u> x-value <u>except</u> $x=3$.

When finding the limit as $x \to 3$, we <u>don't care</u> what actually happens at $x=3$, only nearby.

$$\frac{x^2-9}{x-3}$$



$x \neq 3$

Skipping: Intermediate Value Theorem

Section 1.8: Extending the Idea of a Limit

* One-sided limits

In 1.7: $\lim_{x \to 2} f(x)$: the # that it looks like $f(2)$ should be looking at x-values to the left + right of 2.

In 1.8:

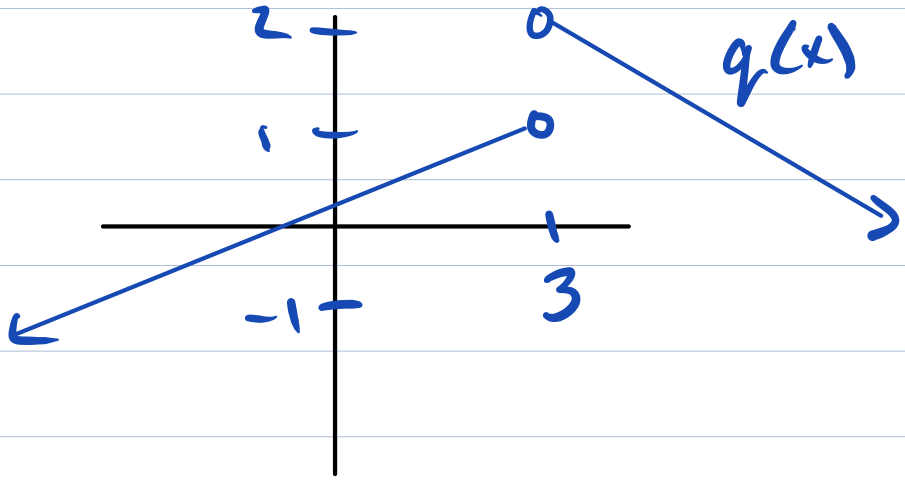
$\lim_{x \to 2^+} f(x)$

$x \to 2$ from the right only

$$\lim_{x \to 2^-} f(x)$$

$x \to 2$ from the left only

Ex:



$g(3)$ DNE

$$\lim_{x \to 3^+} g(x) = 2$$

$$\lim_{x \to 3^-} g(x) = 1$$

$$\lim_{x \to 3} g(x) = \text{DNE}$$

If the limit from the left does not equal the limit from the right, then the normal two-sided limit DNE.

Ex: Find all three kinds of limits at $x = 2$ for $\dfrac{|x-2|}{x-2}$

$|3| = 3$

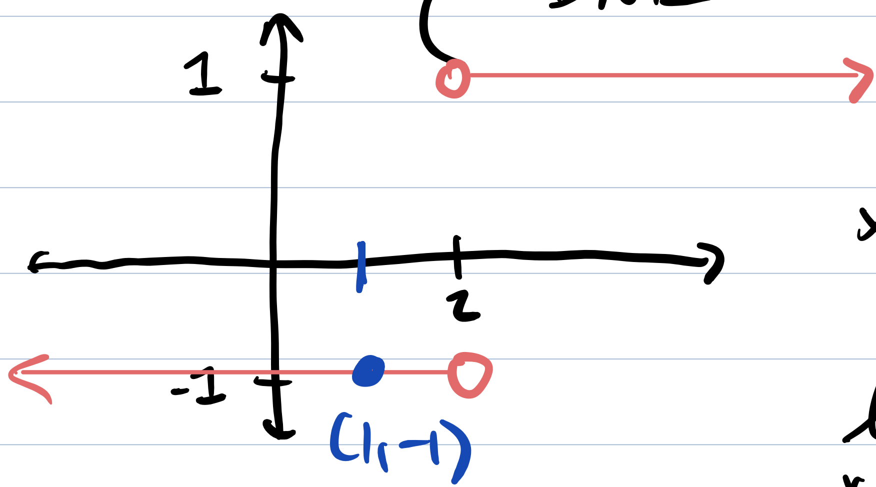
$|-3| = 3$

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} & \text{if } x-2 \geq 0 \\ \frac{-(x-2)}{x-2} & \text{if } x-2 < 0 \end{cases}$$

7

$-1 < 0$

$$= \begin{cases} 1 & \text{if } x-2 > 0 \quad x>2 \\ -1 & \text{if } x-2<0 \quad x<2 \\ \text{DNE} & \text{if } x=2 \end{cases}$$



$$\lim_{x \to 2^-} \frac{|x-2|}{x-2} = -1$$

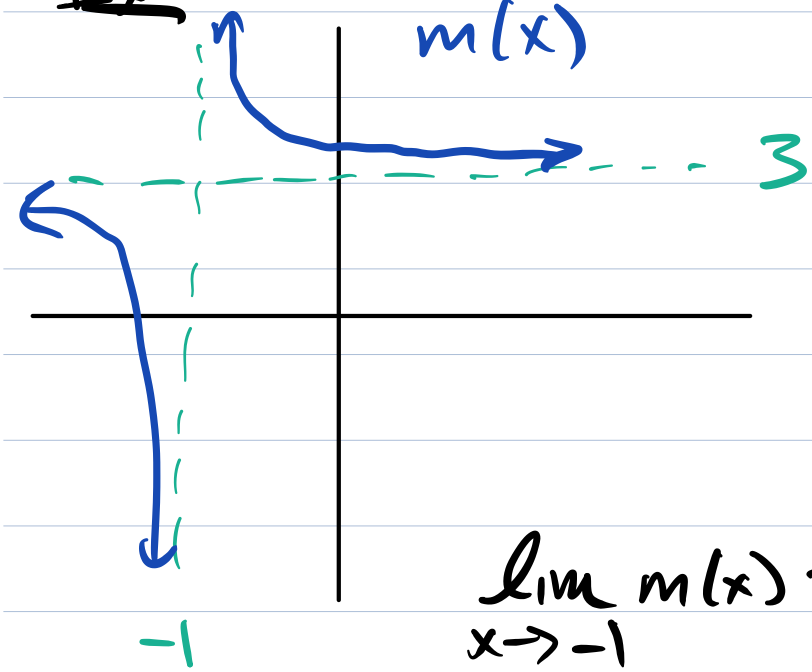
$$\lim_{x \to 2^+} \frac{|x-2|}{x-2} = 1$$

$$\lim_{x \to 2} \frac{|x-2|}{x-2} = \text{DNE}$$

Plug in $x=1$: $\frac{|1-2|}{1-2} = \frac{|-1|}{-1} = \frac{1}{-1} = -1$

$(1,-1)$

Ex:



$$\lim_{x \to \infty} m(x) = 3$$

$$\lim_{x \to -\infty} m(x) = 3$$

$$\lim_{x \to -1} m(x) = \text{DNE}$$

$$\lim_{x \to -1^+} m(x) = +\infty$$

$$\lim_{x \to -1^-} m(x) = -\infty$$