MATH 1450 – EXAM 2

Wednesday, October 26

Name: _____

Circle your Tu/Th discussion time: 8am   9am   10am   11am   12pm   1pm

Scores

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Section 1: True / False. Choose True or False. | **If you choose False, explain briefly why the statement is wrong.**

1. If $\lim\limits_{x \to -2} f(x) = -1$, $\lim\limits_{x \to -2} g(x) = 2$, and $\lim\limits_{x \to -2} h(x) = 4$, then

$$\lim\limits_{x \to -2} \left(g(x) - \frac{h(x)}{f(x)} \right) = 6.$$

              True   False

2. $\dfrac{d}{dx} \left(2x^5 - x^3 \right) = 5x^4 + 3x^2.$

              True   False

3. $\lim\limits_{x \to -1} \dfrac{x}{(1+x)^2} = -\infty$

              True   False

4. If the derivative of $f(x)$ is increasing at $x = c$, then the tangent line at $x = c$ has positive slope.

<center>True   False</center>

5. Let $g(x)$ be a function. If $g'(0)$ exists, then $g(x)$ is continuous at $x = 0$.

<center>True   False</center>

6. If $R(x)$ and $S(x)$ are continuous for all x-values, then so is $R(S(x))$.

<center>True   False</center>

Section 2: Short Answer. You do not need to show your work in this section, but it may help us give you partial credit.

7. For each of the functions $f(x)$ in the top row of the table below, and each of the numbers c in the left column, determine if $f'(c)$ is positive (write "+"), negative (write "−"), zero (write "0"), or if it doesn't exist (write "DNE").

c $\diagdown$ $f(x)$	$x^2 + 4x$	$|x|$	$\dfrac{1}{x}$	2^{-x}
-2				
0				
2				

8. The function $f(x)$ is shown on the left below. On the blank axes on the right, plot $f'(x)$.

3
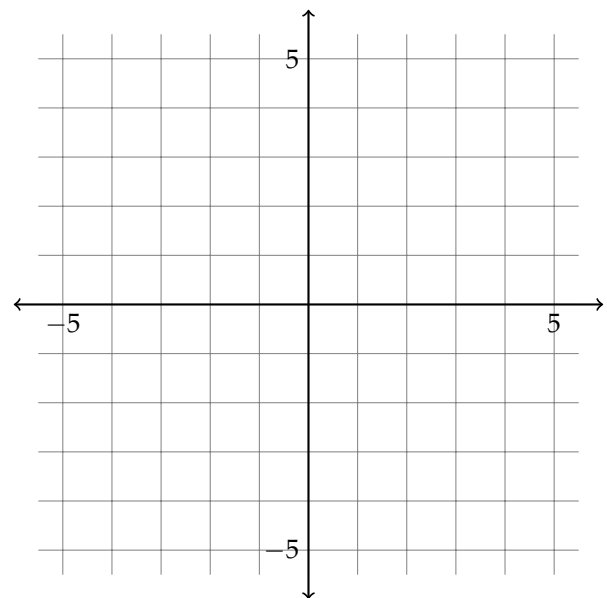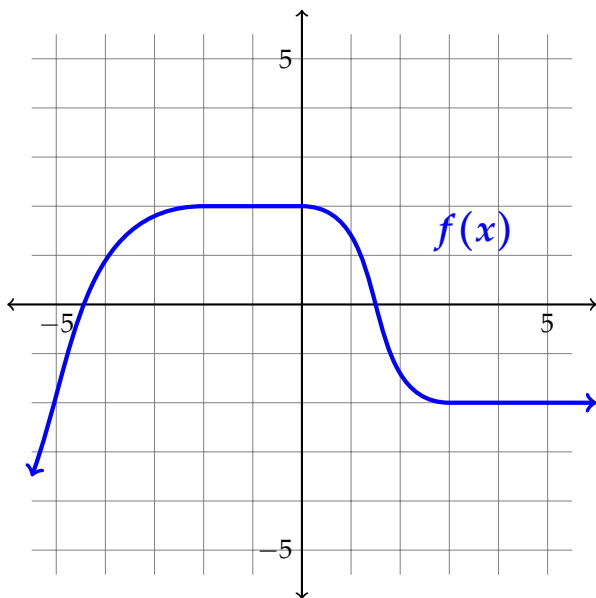
9. Let $T(t)$ denote the temperature outside in Fahrenheit in Milwaukee t hours after noon on a sunny warm July 1. Consider the statement "At 8pm, the rate at which the temperature is dropping is speeding up."

Is $f(8)$:   positive   zero   negative   unknown   (circle one)

Is $f'(8)$:   positive   zero   negative   unknown   (circle one)

Is $f''(8)$:   positive   zero   negative   unknown   (circle one)

Is $f'''(8)$:   positive   zero   negative   unknown   (circle one)

10. The cost of resurfacing a playground of area A square feet is $C(A)$ dollars. Interpret each of the following statements into an English sentence *including a unit of measure*.

(a) $C(140) = 2100$

(b) $C'(140) = 10$

(c) $C''(140) = -1$

Section 3: Long Answer. Answer each question. Your must show your work.

11. On the axes below, draw a function $f(x)$ satisfying ALL of conditions below.

> You may want to draw it on the back of an exam page first, then transfer it to the axes below, so that your final answer is neat and readable.

   ⋆ $f(x)$ is continuous at all x-values.
   ⋆ $f(x)$ is differentiable at all x-values <u>except</u> at $x = 3$
   ⋆ $f(x)$ is positive on the interval $(2, \infty)$
   ⋆ $f(x)$ is negative on the interval $(-\infty, 2)$
   ⋆ $f(x)$ increasing on the intervals $(-\infty, -3)$ and $(0, \infty)$
   ⋆ $f(x)$ is decreasing on the interval $(-3, 0)$
   ⋆ $f(x)$ is concave up on the interval $(-1, 3)$
   ⋆ $f(x)$ is concave down on the intervals $(-\infty, -1)$ and $(3, \infty)$

5
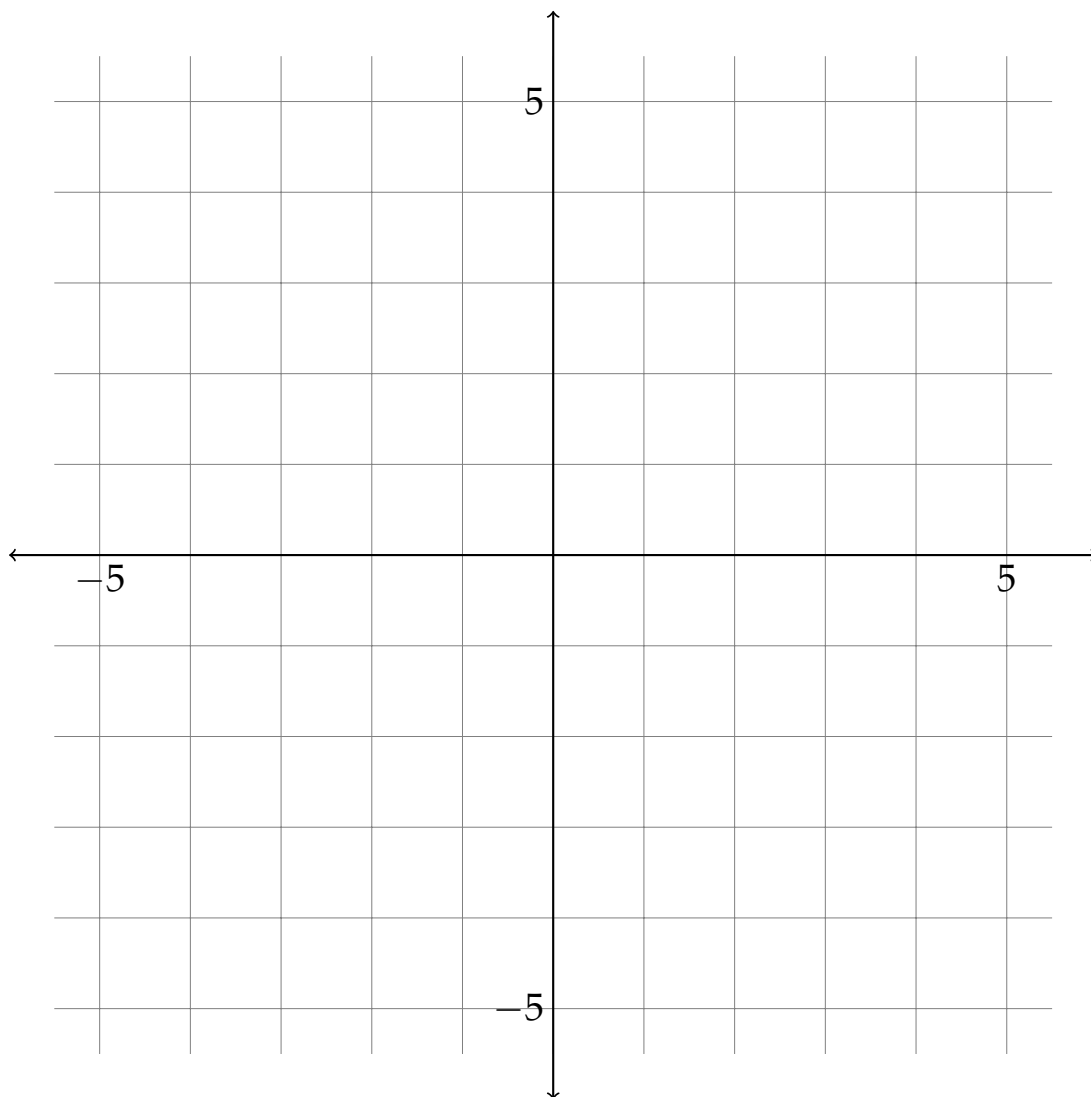
12. Compute each of the limits below. (Make sure to explain your work.)

(a) $\lim\limits_{x \to 0} \left(x^2 \cos\left(\dfrac{1}{x}\right) + 3\cos(x) \right)$

(b) $\lim\limits_{x \to \infty} xe^{-x}$

(c) $\lim\limits_{x \to -\infty} \dfrac{3x^2 - 4x + 1}{2 - x^2}$

13. Let $p(x) = \dfrac{1}{1+x}$.

    (a) Use the limit definition of derivative to find $p'(2)$.

    (b) Determine the equation for the tangent line of $p(x)$ as $x = 2$.

    (c) Use the values of $p(2)$ and $p'(2)$ to estimate $p(2.1)$. (You may leave your answer as an expression involving the sum and/or difference of fractions.)