MATH 1450 – EXAM 1

Wednesday, September 28

Name: _____

Circle your Tu/Th discussion time: 8am 9am 10am 11am 12pm 1pm

Read ALL instructions carefully.

1. You have 50 minutes to complete this exam.
2. **No calculators**, notes, books, or other resources may be used.
3. Please remove all **headphones** and **smartwatches** and put them away.
4. Please silence your **cell phone** and put it away.
5. Keep your eyes on your own paper, and try not to leave your paper easily visible to others.
6. You must show your work and explain all reasoning unless otherwise stated.
7. If I cannot read your writing, you will not receive credit.
8. **Read the instructions of each section.**

Scores

page 1	/6
page 2	/6
page 3	/9
page 4	/9
page 5	/8
page 6	/8
page 7	/7
page 8	/7
Total	/60

The Marquette University honor code obliges students:

- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.
- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

If you have read the instructions above and you understand and agree to abide by the honor code, sign here:

_____

Section 1: True / False. Choose True or False. **If you choose False, explain briefly why the statement is wrong.**

1. For any base $c > 0$, it is possible to rewrite $\log_a(b)$ as

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}.$$

True False

2. $\lim_{x \to -\infty} 2^x = -\infty$

True False

3. There is no solution to the equation $5 \cdot 2^x = 2 \cdot 3^x$ because 3^x grows faster than 2^x.

True False

4. $\lim_{x \to 2} \dfrac{(x^2 + x - 6)\sqrt{x}}{(x^2 - 2x)\cos(\pi x)} = \dfrac{5\sqrt{2}}{2}$.

True False

5. A polynomial of degree N can turn around at most N times.

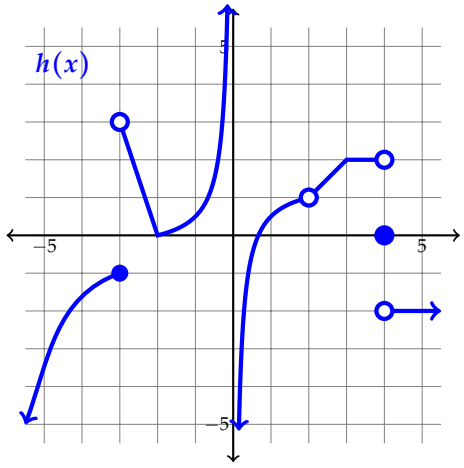
True False

6. For all $a, b, c > 0$, the equation $\log_a(bc) = \log_a(b) + \log_a(c)$ is true.

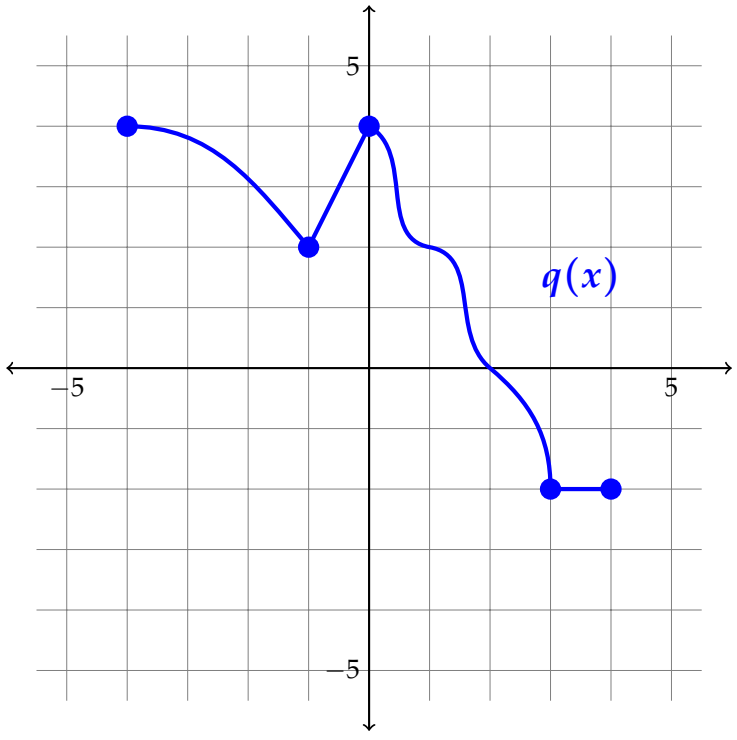
True False

Section 2: Short Answer. You do not need to show your work in this section, but it may help us give you partial credit.

7. Based on the graph below, find the intervals on which $h(x)$ is continuous.

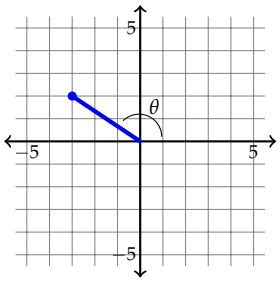


8. The function $q(x)$ is shown on the graph below. Write, in order, the transformations that you must do to $q(x)$ to obtain $-q(x+1)+1$, and then plot this new function on the same axes.



9. Find the equation for the line that passes through $(-1, -2)$ and is parallel to the line $3x - y = 1$

10. Consider the angle θ show below, formed by the line drawn and the positive x-axis.



(a) What is $\tan(\theta)$?

(b) Is $\sin(\theta)$ positive, negative, or zero? Is $\cos(\theta)$ positive, negative, or zero?

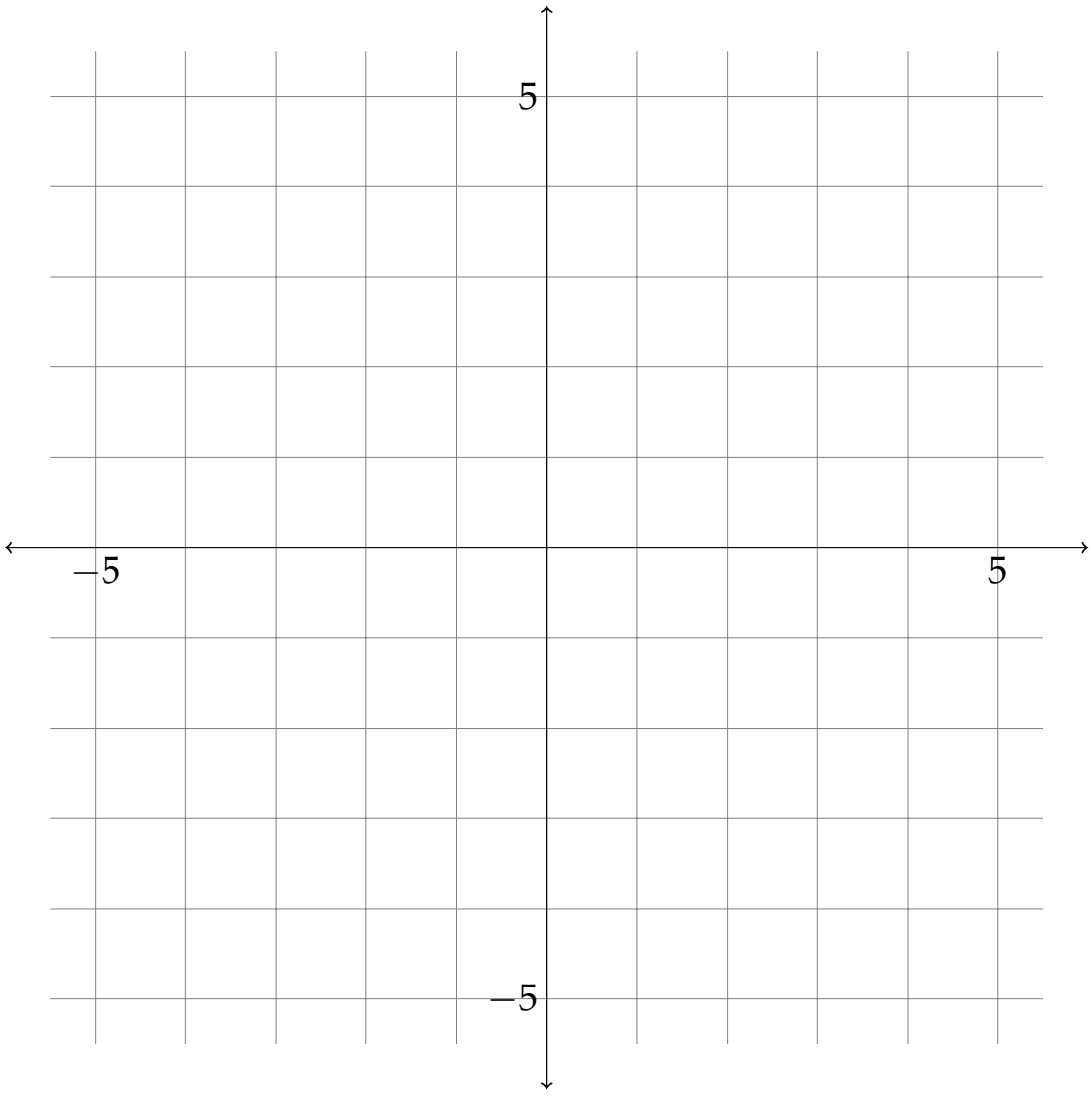
(c) What is $\sin^2(\theta) + \cos^2(\theta)$?

Section 3: Long Answer. Answer each question. Your must show your work.

11. On the axes below, draw a function $f(x)$ satisfying ALL of conditions below.

    You may want to draw it on the back of an exam page first, then transfer it to the axes below, so that your final answer is neat and readable.

    - ⋆ The domain is all real numbers.
    - ⋆ The range is all real numbers.
    - ⋆ $\lim_{x\to-\infty} f(x) = 3$
    - ⋆ $\lim_{x\to-3^-} f(x) = -\infty$
    - ⋆ $\lim_{x\to-3^+} f(x) = 0$
    - ⋆ $\lim_{x\to-1} f(x) = 2$
    - ⋆ $f(-1) = 5$
    - ⋆ $\lim_{x\to1} f(x)$ does not exist
    - ⋆ $\lim_{x\to3} f(x) = 0$
    - ⋆ $f(3) \neq 0$
    - ⋆ $\lim_{x\to\infty} f(x) = 0$



12. A scientist is growing two colonies of bacteria in separate petri dishes. Colony A is a strain that doubles its population every hour, and the scientist starts this colony with 100 bacteria at 12:00pm. Colony B is a strain that triples its population every hour, and the scientist starts this colony with 20 bacteria at 3:00pm.

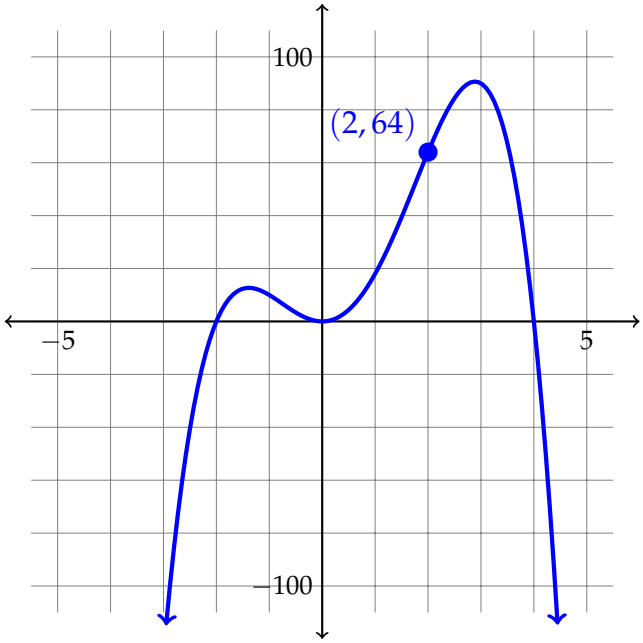
    (a) Find functions $A(t)$ and $B(t)$ that model the populations of Colonies A and B, where t is the number of hours after 12:00pm on the day they were started.

    (b) What is the population of each colony at 12:00pm the following day?

    (c) How many hours after 12pm on the day they're started will the population of the two colonies be equal?

    (d) Compute $B^{-1}(t)$. What does the function $B^{-1}(t)$ represent? (What does it tell you in terms of bacteria, hours, etc.?)

13. Find a polynomial of degree four that matches the graph below.



14. Define $f(x) = \dfrac{1}{x-2}$ and $g(x) = \sqrt{2x+4}$.

(a) Compute $f(g(x))$ and state its domain.

(b) Compute $g(f(x))$ and state its range.

(c) If two functions $A(x)$ and $B(x)$ both have an inverse, is it true that their composition $A(B(x))$ also has an inverse? If so, explain what that inverse is; if not, give an example of invertible functions $A(x)$ and $B(x)$ for which $A(B(x))$ is not invertible.