MATH 2100 / 2350 – HOMEWORK 3

Fall 2020

due Wednesday, **October 7**, on D2L, by the beginning of class

Sections 3.1, 3.2

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due on D2L at the *beginning* of class. It must be typed in Latex (other formats such as Word are not acceptable). **You must submit the .pdf file, but you do not have to submit the .tex file unless I ask for it** Any pictures can be drawn by hand and added to the Latex file with the "\includegraphics" command (see how I do it in this document). Please write the questions in the correct order. Explain all reasoning.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be the convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write "$3 \in S \implies 3 \notin \overline{S}$"; instead, write "Since $3 \in S$, it follows that $3 \notin \overline{S}$".

1. Use Venn Diagrams to determine whether the equation below is true:
$$(B \cup (A \smallsetminus C)) \cap A \quad = \quad A \smallsetminus (A \cap \overline{B} \cap C)$$

2. Use Venn Diagrams to determine whether the equation below is true:
$$(\overline{A \cup B}) \cup (\overline{A \cup C}) \quad = \quad (B \cup C) \smallsetminus A$$

3. List 5 elements of each of the following sets, unless there are fewer than 5 elements (in which case, list them all and justify how you know you've listed all of them).

   (a) $\{x \in \mathbb{R} : x \notin \mathbb{N} \text{ and } x^2 \in \mathbb{N}\}$

   (b) $\{S \subseteq \mathbb{N} : \text{the sum of the elements in } S \text{ is less than } 3\}$

   (c) $\{z \in \mathbb{N} : z = 5k + 2 \text{ for some } k \in \mathbb{Z}\}$

   (d) $\{r \in \mathbb{Z} : r = 2k \text{ for some } k \in \mathbb{Z} \text{ and } r = 2\ell + 1 \text{ for some } \ell \in \mathbb{Z}\}$

4. Write each of the following sets in set-builder notation.

   (a) The set A of real numbers that are not rational numbers.

   (b) The set B of rational numbers whose numerator is 1 and whose denominator is a prime number.

   (c) The set C of pairs of real numbers (r_1, r_2) that add up to a natural number.

   (d) The set D of subsets of the real numbers whose size is 10 or less.

5. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

For all sets A, B, and C: if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

6. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

For all sets A and B: $(A \times A) \smallsetminus (B \times B) = (A \smallsetminus B) \times (A \smallsetminus B)$.

7. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

For all sets A and B: if $A \subseteq B$ then $A \subseteq \mathcal{P}(B)$.