MATH 2100 / 2350 – EXAM 1

Wednesday, October 2

Name: Key

Instructions: Please write your work neatly and clearly. **You must explain all reasoning. It is not sufficient to just write the correct answer.** You have 75 minutes to complete this exam. You may not use calculators, notes, or any other external resources.

Scores

1	
2	
3	
4	
5	
6	
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10	

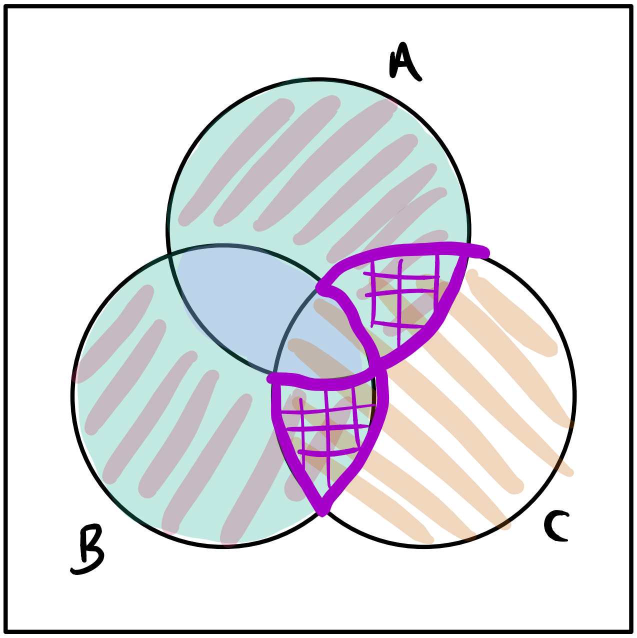
The Marquette University honor code obliges students:

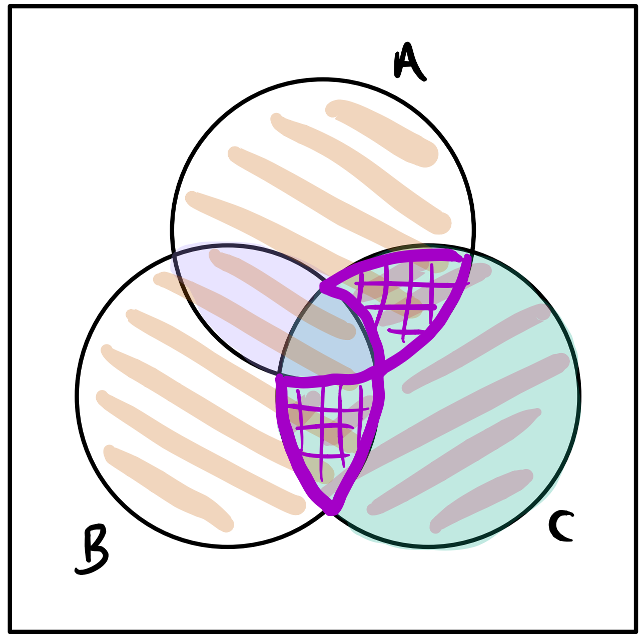
- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.
- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

If you understand and agree to abide by this honor code, sign here:

1. Use Venn Diagrams to decide whether the equation below is true.

$$\Big((A \cup B) \setminus (A \cap B)\Big) \cap C \;=\; \Big(C \setminus (A \cap B)\Big) \cap (A \cup B)$$





So, yes, the sets are equal.

2. Decide whether each of the following statements is true or false for finite sets A and B. If false, give a counterexample (make it as simple as possible!). If true, no justification is needed.

   Recall that given a set S, "$|S|$" means "the size of S". The book uses $n(S)$ to mean the same thing.

   (a) $|A \times B| = |A| \cdot |B|$

   True   False

   (b) $|A \cup B| = |A| + |B|$

   True   False

   $A = \{1\}$
   $B = \{1\}$ $\Rightarrow A \cup B = \{1\} \Rightarrow |A \cup B| = 1$

   but $|A| + |B| = 2$

   (c) $|A \setminus B| = |A| - |B|$

   True   False

   $A = \{1\}$ $\quad |A \setminus B| = |\{1\}| = 1$
   $B = \{2\}$ $\quad |A| - |B| = 1 - 1 = 0$

   (d) If $|A| = |B|$, then $|\mathcal{P}(A)| = |\mathcal{P}(B)|$.

   True   False

3. Define T to be the set of positive integers N that satisfy the following property:

If N is a perfect square, then it is an even number.

Write the set T in set builder notation.

$$T = \{N \in \mathbb{Z}^+ : (N = k^2 \text{ for some } k \in \mathbb{N}) \rightarrow (N = 2\ell \text{ for some } \ell \in \mathbb{N})\}$$

(many possible variations)

4. List five elements in each of the following sets, unless there are fewer than 5 elements in the set (in which case, list all of them and justify how you know you've listed all of the elements).

   (a) $\{q \in \mathbb{N} : q^2 - q \text{ is even and } q \text{ is a perfect square}\}$

   (answers may vary)

   0, 1, 4, 9, 16 (all perfect squares work!)

   (b) $\{T \in \mathbb{Q} : T \notin \mathbb{N} \text{ and } 10T \in \mathbb{N}\}$

   $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}$

   (c) $\{X \in \mathcal{P}(\{\heartsuit, \diamondsuit, \clubsuit, \spadesuit\}) : |X \smallsetminus \{\heartsuit\}| = 1\}$

   $\{\diamondsuit\}, \{\clubsuit\}, \{\spadesuit\}$

   $\{\diamondsuit, \heartsuit\}, \{\clubsuit, \heartsuit\}$

5. Consider the statement below.

There is a textbook that no teachers like.

(a) Convert the statement from English to math using quantifiers and predicates. Your predicate should have *no quantifiers in it*.

Let B = set of all textbooks

T = set of all teachers

$P(t,b)$ = "teacher t does not like textbook b"

$$\exists b \in B, \forall t \in T, P(t,b)$$

(b) Negate the statement mathematically.

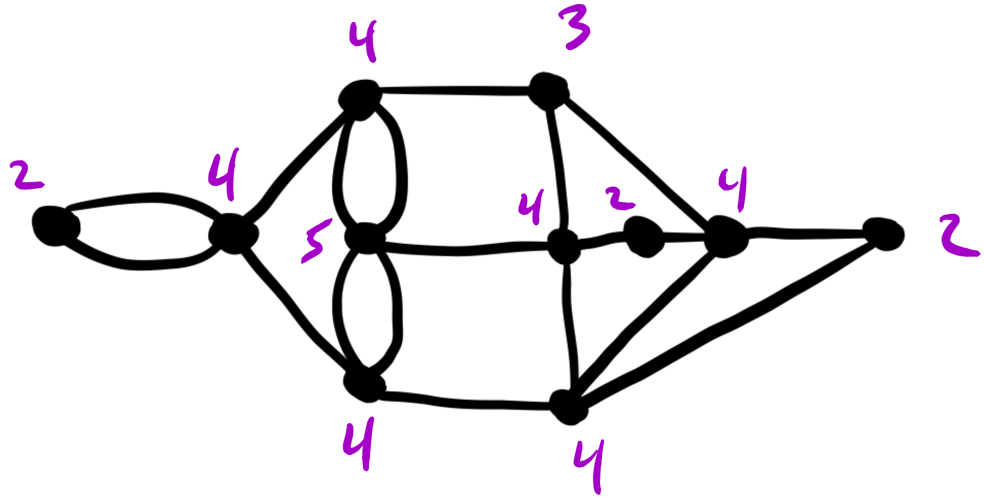
$$\forall b \in B, \exists t \in T, \neg P(t,b)$$

(c) Convert the negation back to English.

For all textbooks, there is at least one teacher that likes it.

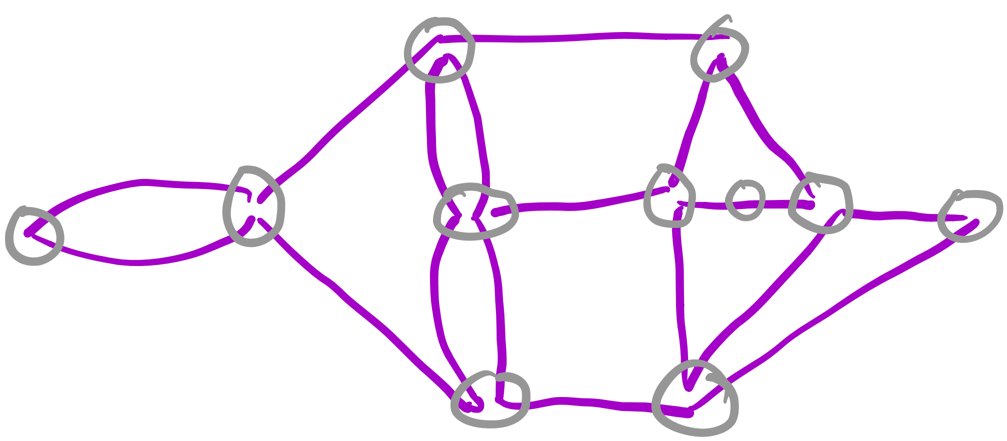
6. For the graph below, find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit. (Note, if they exist, you must actually find one, not just show that one exists.)

Degrees:



Since there are vertices of odd degree, there is no Eulerian circuit.

An Eulerian path is:



7. Consider the fact below. If it is true, give a brief explanation why. If it is false, give a specific counterexample.

For all sets A, B, and C, if $(A \cup B) \subseteq (A \cup C)$, then $B \subseteq C$.

False

Counterexample: $A = \{1\}$, $B = \{1\}$, $C = \emptyset$

Then

$A \cup B = \{1\}$, $A \cup C = \{1\}$, so $(A \cup B) \subseteq (A \cup C)$,
but it is not true that $B \subseteq C$.

8. Consider the implication

$$R = \text{"If it is raining, then your car's headlights are on."}$$

(a) What is the converse of R?

If your car's headlights are on, then it's raining.

(b) What is the contrapositive of R?

If your car's headlights are off, then it is not raining.

(c) What is the inverse of R?

If it is not raining, then your cars headlights are off.

(d) Which of the above 3 statements is/are logically equivalent to R?

A statement is always logically equivalent to its contrapositive.

9. Use a truth table to determine whether the following logical equivalence is true.

$$(r \to (s \vee t)) \wedge (s \to t) \quad \equiv \quad r \to t$$

r	s	t	$s \vee t$	$r \to (\ )$	$s \to t$	$(\ ) \wedge (\ )$	$r \to t$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	F	F
T	F	T	T	T	T	T	T
T	F	F	F	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

Not equal, so not logically equivalent.

10. (a) You find three people, Jay, Kay, and Elle, who say the following:

Jay: If I'm lying, then so is Kay. $(\neg J) \to (\neg K)$

Kay: Either Jay is lying or Elle is lying, but not both. $((\neg J) \vee (\neg L)) \wedge (J \vee L)$

Elle: If Kay is lying, then the moon is made of cheese[1]. $(\neg K) \to F$ (implies $K \equiv T$)

Who is telling the truth and who is lying?

false when $J \equiv F$ and $K \equiv T$

J	K	L	Jay's stmt	Kay's stmt	Elle's stmt	
T	T	T	T	F	T	x
T	T	F	T	T	T	x
T	F	T	T	F	F	x
T	F	F	T	T	F	x
F	T	T	F	T	T	✓
F	T	F	F	F	T	x
F	F	T	T	T	F	x
F	F	F	T	F	F	x

Only one possibility: Jay is lying, Kay + Elle are telling the truth.

(b) What if you had the same three people, but Jay says "I am lying", and nobody else says anything? Can you conclude anything?

This would immediately be a contradiction, implying no solutions.

J	Jay's stmt
T	F
F	T

[1] For the purposes of this question, you may assume that the moon is not made of cheese.